College- and Career-Readiness Standards for Mathematics

Exemplar Lesson Plan

"Are We Related, or Just a Function?"

F-IF.1

F-IF.2

F-IF.5

F-IF.6

MISSISSIPPI
DEPARTMENT OF
EDUCATION
Ensuring a bright future for every child

MISSISSIPPI DEPARTMENT OF EDUCATION

Ensuring a bright future for every child

Carey M. Wright, Ed.D., State Superintendent of Education

Office of the Chief Academic Officer

Kim S. Benton, Ed.D., Chief Academic Officer

Office of Professional Development

Trecina Green, Executive Director

Carol Ladner, Professional Development Coordinator

Elizabeth Fulmer, Professional Development Coordinator

Office of Secondary Education

Jean Massey, Executive Director

Marla Davis, Ph.D., NBCT, Bureau Director

Office of Elementary Education and Reading

Nathan Oakley, Executive Director

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

COURSE: ALGEBRA I	
Title: ***Are We Related or Just a Function?*** **Estimated Duration:** 3 or 4 days	**Real World Purpose:** A large part of the world around us runs by mathematical rules. Functions are the heart of many mathematics courses, starting with the Algebra I course and moving forward. Functions are important because they are the mathematical building blocks for designing plants, finding cures for diseases, forecasting world disasters, understanding global economics, and manufacturing consumer products, just to name a few.

I Can:

F-IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.

F-IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

F-IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Prerequisite Skills:

- Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. **(8.EE.5)**
- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. **(8.EE.6)**
- Solve linear equations in one variable.
  b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **(8.EE.7b)**
- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **(8.F.1)**
- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **(8.F.4)**
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **(8.F.5)**

Materials/Resources:

- www.everythingmaths.co.za
- www.saylor.org
- www.illustrativemathematics.org
- www.cobbk12.org
- www.cutegraphics.com
- www.clker.com
- www.geocities.ws
- www.onemathematicalcat.org
- Algebra tiles
- Mini-white board
- X-Y Coordinate Geoboards
- Attachments (Total: 10)
- White board
- Mini-white board with x-y coordinate plane

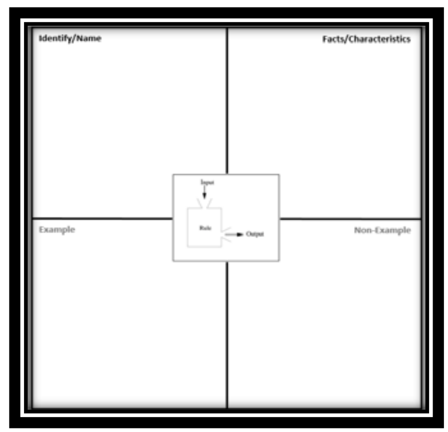
Key Vocabulary:

- Function
- Domain
- *f(x)* notation
- Average rate of change
- Relation
- Mapping Diagram
- Ordered pairs
- Function Rules
- Vertical line test
- Range
- Output
- Input
- Equation
- Independent variable
- Dependent variable
- Function notation
- Set notation
- Secant line

Lesson Introduction

Student Exploration Activity:

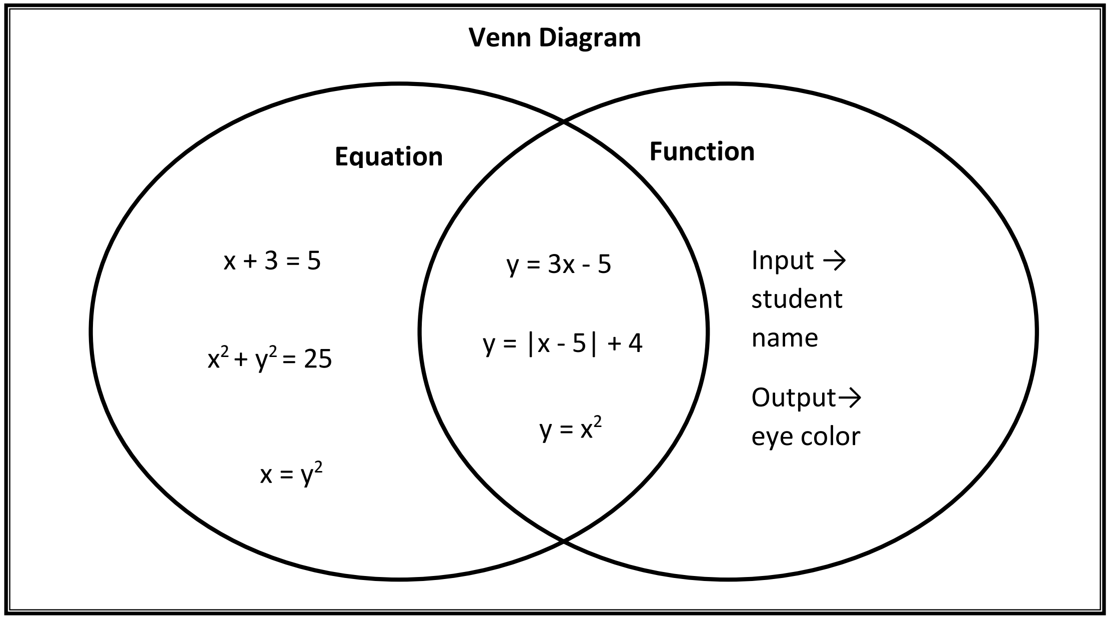
Part 1 (8.F.1) – Are All Equations a Function?



Frayer Model for Function Input and Output, Attachment #1

Students will be given the above Frayer Model at the beginning of class. Students will individually complete the Frayer Model using prior knowledge from 8.F.1. Students will only have seven minutes to complete as much of the model as possible. Students will then form groups of four. In their groups, students will compile all individual Frayer Models into one that best represents their group and display it on chart paper. One student from each group will be chosen to give a presentation about their model while the other groups do a gallery walk. The top three models will be displayed in the Gallery Walk Hall of Fame.

Remaining in groups of fours, the teacher will direct students to take out their mini -white boards. Teacher will ask students to create an equation, if possible, and if not already done by the original group, students will create equations for the top three examples given in the Frayer Models. Teacher will review the equations created by the students, calling upon different groups to present their equations to the class. Teacher will pose the following question to the class, "Are all equations functions?" Teacher will give the students a chance to answer this question by continuing group work. Teacher will circulate the classroom monitoring the progress of students. Teacher will ask probing questions to encourage students to explore the definition of a function using different models for the equations. If any students are able to create an equation(s) that is not a function, the teacher will engage the whole class in a discussion about that equation. The discussion may include creating a table of data for the equation, showing an input and output model for the equation, and possibly even graphing the equation on the smartboard as a visual model for the equation. If the class is not able to answer the question "Are all equations functions?" through student exploration, then the teacher will pose the following equation for the class: $x^2 + y^2 = 25$. Teacher will graph the equation and explain that the equation is the standard form for a circle. Teacher will wrap up the activity by having students create the following Venn Diagram on a notecard/index card to keep with them throughout this Unit. The Venn Diagram explains the relationship between equations, functions, and inputs and outputs.



An equation can have an input without that input having a unique output. Therefore, all equations are not functions. A function can have an input with a unique output without being an equation. Therefore, all functions are not equations. Some equations will have inputs with each having unique outputs and will therefore also be classified as functions.

Lesson Activities

Day 1

1. **Activity #1: 8.F-1 and F-IF.1**
   Relation Vocabulary 4-Part Foldable, Teacher Guided Instruction and Student Independent Work (10 minutes estimated)
   Teacher will distribute the ***Relation Vocabulary 4-Part Foldable*** (Attachment #2). Students will complete the foldable as the teacher discusses the following vocabulary words. Upon completion, students can cut out the new vocabulary words and place in an envelope titled for this section. Teacher can use the 4-Part Foldable technique with other vocabulary words throughout the lesson as needed.

Relation: a set of ordered pairs
Domain: the set of input values (*x*) in a relation; *x* is also called the independent variable
Range: The set of output values (*y*) in a relation; *y* is also called the dependent variable
Independent variable: the variable in a relation with a value that is subject to choice
Dependent variable: the variable in a relation with a value that depends on the value of the independent variable

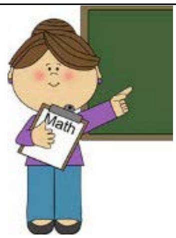
2. **Activity #2: F-IF.1**
Determining Domain and Range of a Relation, Teacher Guided Instruction and Student Pair and Share (30 minutes estimated)

__Question__: State the domain and range of the following relations. List them from least to greatest using set notation.

a. {*(9,3), (-1, 4), (0, 5), (-6, -3), (10,12)*}
b. {*(7,0), (2, -8), (-11, 3), (2, 8), (1,-6)*}
c. {*(0,1), (-3, 6), (3, 5), (-13, 14), (7,9)*}

__Solutions__

a. Domain { *-6, -1, 0, 9, 10* } Range { *-3, 3, 4, 5, 12* }
b. Domain { *-11, 1, 2, 5, 7* } Range { *-8, -6, 0, 3, 8* }
c. Domain { *-13, -3, 0, 3, 7* } Range { *1, 5, 6, 9, 14* }



<u>Quesiton:</u> Hotdogs at the ballpark cost *$1.50*, write the relation, in set notation, that shows the purchase of one, two, and three hotdogs. What would five hotdogs cost? Expand the relation to determine the cost of five hotdogs. Can you write a rule to determine the cost of any number of hotdogs? Identify the dependent and independent variables.

<u>Solutions</u>
One hotdog costs *$1.50*, two hotdogs cost *$3.00*, three hotdogs cost *$4.50*
{*(1, 1.50), (2, 3.00), (3, 4.50)*}

Four hotdogs cost *$6.00*, five hotdogs cost *$7.50*
{*(1, 1.50), (2, 3.00), (3, 4.50), (4, 6.00), (5, 7.50)*}

Cost equals *$1.50* (per hotdog) multiplied by the number of hotdogs purchased.
C = $1.50 x n
C is the independent variable, cost
n is the dependent variable, number of hotdogs

<u>Question:</u> What are at least two other names for *cost*, the independent variable? What are at least two other name for *number of hotdogs*, the dependent variable?

<u>Solutions:</u>
Cost: independent variable, *y* variable, range, or output
Number of hotdogs: dependent variable, *x* variable, domain, or input

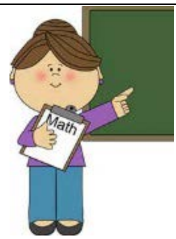
The "*independent variable*" causes the "*dependent variable*" to change!

Teacher will distribute flashcards from ***Dependent/Independent Flashcards*** (Attachment #3) to student pairs. Student pairs will display the appropriate flashcard in response to the statements /questions read by the teacher to the whole class. The statements/questions can be found in ***Am I Dependent or Independent?*** (Attachment #4). Student pairs will challenge others' responses through debate and examples of mathematical rules that support their claim (for problems #3 - #5 only). Teacher will review any student misunderstandings by asking probing and/or thought provoking questions.

3. **Activity #3: F-IF.1**
   Determining if a Relation is a Function, Teacher Guided Instruction and Student Pair and Share (50 minutes estimated)

   A function is a special type of relation in which each input has exactly one output. Functions can be represented by the following: ordered pairs, table of values, mapping diagrams, graphs (vertical line test), and function notation. Students will display these representations in a bubble map using the ***Bubble Map Template*** (Attachment #5).

   Ordered Pairs
   A given relation is a function if each input (*x* value) is paired with exactly one output (*y* value). The *x* values should not repeat.



<u>Question</u>: Is either relation below a function? Explain why or why not? If a function, state the domain and range.

A. {*(2, 4), (-5, 7), (3, 6), (-5, 3), (0, -2)*} B. {*(6, -1), (-3, 5), (2, 4), (9, 0), (-7, 1)*}

<u>Solutions:</u>

A. {*(2, 4), (-5, 7), (3, 6), (-5, 3), (0, -2)*} is not a function because the input -5 has two output values.

B. {*(6, -1), (-3, 5), (2, 4), (9, 0), (-7, 1)*} is a function because each input has exactly one output.

Domain {*-7, -3, 2, 6, 9*} Range {*-1, 0, 1, 4, 5*}

Table of Values

A given table of values is a function if each input (*x* value) is paired with exactly one output (*y* value). The *x* values should not repeat.

<u>Question:</u> Which input-output table below represents a function? Explain why or why not? If a function, state the domain and range.

Table A	
Input	**Output**
-5	-3
0	8
3	1
0	4
7	2

Table B	
Input	**Output**
-2	8
1	0
6	-3
7	2
15	-3

Solutions:

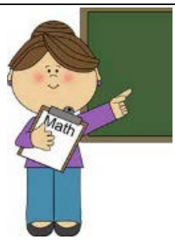
Table A is not a function because the input value 0 has two output values.

Table B is a function because each input value has exactly one output value.

Domain {-*2, 1, 6, 7, 15*} Range {-*3, 0, 2, 8*}

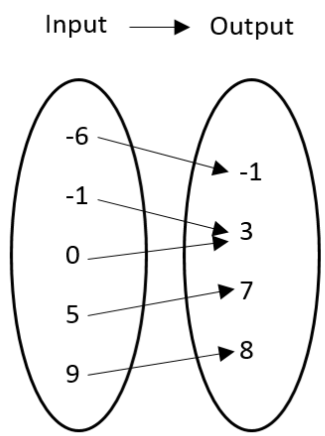
Mapping Diagrams

A mapping diagram is a type of flow chart. It consists of two parallel columns. The first column has the input or *x* values and the second column has the output values or *y* values. An arrow is drawn from each element in the first column to its corresponding element in the second column. A given mapping diagram is a function if each input (*x* value) is paired with exactly one output (*y* value). The *x* values should not repeat.

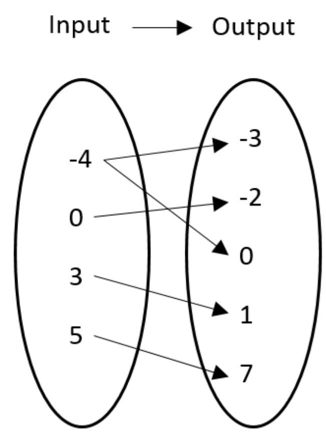


Question: Which mapping diagram below represents a function? Explain why or why not? If a function, state the domain and range.

A



B



Solutions:

Mapping Diagram A is a function because each input value has exactly one output.

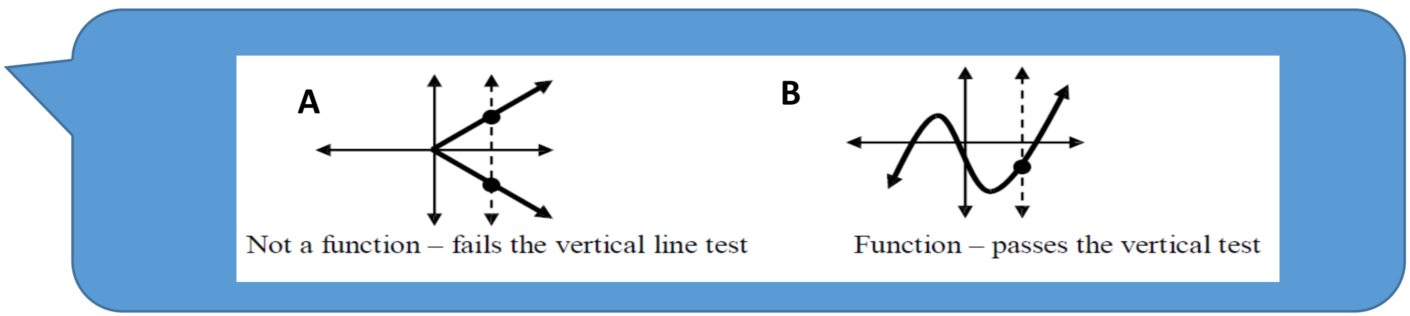
Domain {*-6, -1, 0, 5, 9*} Range {*-1, 3, 7, 8*}

Mapping Diagram B is not a function because the input value -4 has two output values.

Graphs of Functions

Given a graph, the vertical line test can be used to determine if a relation is a function. The vertical line test states that if a vertical line is drawn anywhere on the graph and it does not touch the graph in more than one point, then the graph represents a function. If a vertical line can be drawn between any two points on the graph, then it fails the vertical line test. The two points would have the same *x* value but different *y* values. This particular input (*x*) value has more than one output (*y*) value.

<u>Questions:</u> Which graph below represents a function? Explain why or why not.



Teacher will distribute ***Ways to Represent a Function Poster Project*** (Attachment #6) to student pairs. Teacher will review the guidelines and the rubric with the students. Students will have the remainder of the class period to complete their poster. Teacher will circulate the classroom answering students' questions and guiding students' work. Teacher will review any student misunderstandings by asking probing and/or thought provoking questions.

<u>Day 2</u>

1. **Activity #1: F-IF.1 and F-IF.2**
   Functions and Function Notation, Teacher Guided Instruction and Student Independent Work (20 minutes estimated)

   The functions from yesterday had an input values that exactly one output value. Think back to the Student Exploration Activity and the Venn Diagram created. A function can also be an equation that describes a mathematical relationship between the independent (*x*) variable and the dependent (*y*) variable.

<u>Question</u>: The set {(-1, -1), (0, 1), (1, 3), (2, 5), (3, 7)} is a function and can be expressed by the equation $y = 2x + 1$. This function rule can be used to determine other ordered pairs. Input the value *4* into the function rule and determine the output (y) value it is paired with. Write as an ordered pair and state its significance.

<u>Solutions:</u>

$y = 2x + 1$ The ordered pair is *(4, 9)* and is a solution to the function rule.

$y = 2(4) + 1$

$y = 8 + 1$

$y = 9$

Function Notation

Function notation is another way to name a function that is defined by an equation. For equations written in terms of *x* and *y*, the symbol $f(x)$ replaces *y* and is read as "the value of the function at *x*" or "*f* of *x*".

$y = f(x)$ *y* and the function of *x* are interchangeable

Using the prior example, the equation $y = 2x + 1$ can also be written as $f(x) = 2x + 1$ since y and $f(x)$ are interchangeable and the equation is also a function.

Function notation is useful because it relates the function rule to its graph. Each ordered pair or solution to the function rule is a point on the graph of that function. Function notation allows the ordered pairs to be easily seen.

Evaluate using Function Notation
While parentheses in mathematical expressions and equations typically mean multiplication, the parentheses in function notation does not mean multiply. The expression $f(x)$ mean "evaluate the function for the given value of x". The expression $f(x)$ does not mean "multiply f and x".

Let's compare two methods:

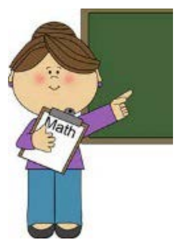
Evaluate the equation for $x = 2$	Evaluate the function for $f(2)$
$y = 3x + 4$	$f(x) = 3x + 4$
$y = 3(2) + 4$	$f(2) = 3(2) + 4$
$y = 6 + 4$	$f(2) = 6 + 4$
$y = 10$	$f(2) = 10$

Notice how the solution, written as an ordered pair *(2, 10),* is easier to see when using function notation.

Teacher will review procedural skill and fluency with function notation and evaluating single functions before introducing function notation for multiple functions. Individually, students will use mini-white boards to answer the questions posed by the teacher from ***Function Notation Review*** (Attachment #7). Teacher can also use this activity as an assessment to evaluate individual student progress and mastery of the standards. Teacher will review any student misunderstandings by asking probing and/or thought provoking questions.

2. **Activity #2: F-IF.2**
Function Notation, Teacher Guided Instruction, and Student Independent Work (20 minutes estimated)

Function notation does not always have to be expressed with the letter *f*. Any letter can be used to represent a function. Other acceptable forms of function notation are *g(x), h(x),* or *k(x),* just to name a few. When more than one function or its graph is being referenced or compared, there is a need to be able to distinguish the function.



Question:
Given: $f(x) = 2x - 11$ $\quad g(x) = 2$ $\quad h(x) = \frac{1}{2}x + 5$ $\quad k(x) = -8x + 4$, *evaluate* the following expressions:
$f(-3)$ $\quad g(10)$ $\quad h(-6)$ $\quad k(0)$

Solutions:

$f(-3) = 2(-3) - 11$	$g(10) = 2$	$h(-6) = \frac{1}{2}(-6) + 5$	$k(0) = -8(0) + 4$
$f(-3) = -6 - 11$	$g(10) = 2$	$h(-6) = -3 + 5$	$k(0) = 0 + 4$
$f(-3) = -17$	$g(10) = 2$	$h(-6) = 2$	$k(0) = 4$

Question:
Function notation can be used to evaluate expressions in the following form:
$f(x) - g(x)$ or $f(x) * g(x)$ or $f(x) + g(x)$ given $f(x) = 2x - 11$ and $g(x) = 2$
Can you evaluate each of them?

Solutions:

f(x) – g(x)	*f(x) ∗ g(x)*	*f(x) + g(x)*
2x – 11 – 2	*(2x – 11)2*	*2x – 11 + 2*
2x – 13	*4x – 22*	*2x – 9*

Teacher will review any student misunderstandings by asking probing and/or thought provoking questions. Students will have an opportunity to summarize their understanding of functions and function notation before Activity #3.

3. **Activity #3: F-IF.2**
Function Notation, Student Pair and Share (50 minutes estimated)

Students will pair to complete ***Function Notation Activity Packet*** (Attachment #8). Teacher will circulate the classroom answering student questions and guiding student work. During the last 10 minutes, the teacher will review the activity for correctness by asking for student responses to questions and allowing students to critique their peers. Teacher will be able to gauge student understanding and mastery through this questioning. Teacher will correct major student misunderstandings through a quick reteach.

Day 3

1. **Activity #1: F-IF.1 and F-IF.2**
Synonyms for Input and Output (Vocabulary Review), Student Independent Work (15 minutes estimated)

At the beginning of class, the teacher will ask students to get their individual mini-white boards and make a T-chart with input at the top of one column and output at the top of the other column. Teacher will instruct students that they have five minutes to write synonyms for the terms input and output using the instruction and activities from the past two days. After five minutes, the teacher will ask all students to display their white boards for a quick informal assessment. Teacher will ask two or three students to present

their t-chars for a whole class discussion. Teacher will guide the discussion using questioning to ensure that all students have the following synonyms for the terms input and output.
Input: domain, dependent (variable or values), *x* value

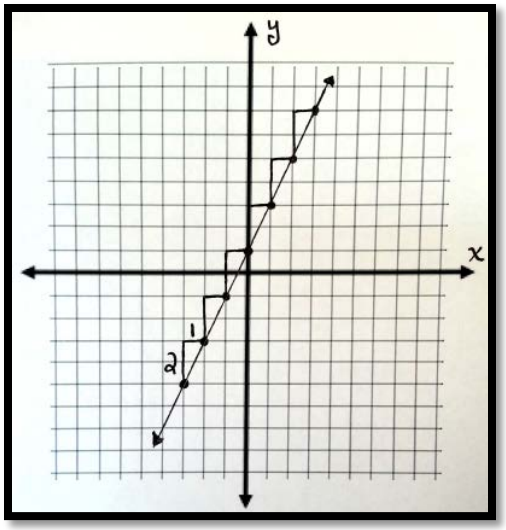
Output: range, independent (variable or values), *y* value, *f(x)*

2. **Activity #2: F-IF.5**
Slope of a Linear Function, Teacher Guided Instruction and Student Pair and Share (35 minutes estimated)

If a relation is a function, it can be written in function notation. Function notation represents a rule that relates the domain of a function to its range. Function notation is where $y = f(x)$ and x is the domain and $f(x)$ is the range.

The function $y = 2x + 1$ has the following table of values and graph.

x	***f(x)***
-3	-5
-2	-3
-1	-1
0	1
1	3
2	5
3	7



The function $y = 2x + 1$ is a linear function because of the constant rate of change. The slope or constant rate of change was found from the graph using similar triangles and prior knowledge from 8.EE.6.

Rate of change Formula

The terms slope and rate of change can be used interchangeably. Slope can also be represented by the variable *m*. Slope can be found without a graph using the slope or rate of change formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Using two points from the table above and the rate of change formula, the slope can be calculated without the aid of the graph. Let the two points be $(x_1, y_1)(-3, -5)$ and $(x_2, y_2)(-2, -3)$.

$$m = \frac{-3 - -5}{-2 - -3}$$
$$m = \frac{2}{1}$$
$$m = 2$$

The slope tells how the dependent variable is changing in relation to the independent variable. There are four types of slope: positive, negative, zero, and undefined. A positive or increasing slope occurs when as *Δx* increases *Δy* also increases. A negative or decreasing slope occurs when as *Δx* increases *Δy* decreases. A zero slope (horizontal line) occurs when *Δy = 0*. An undefined slope (vertical line) occurs when *Δx = 0.*

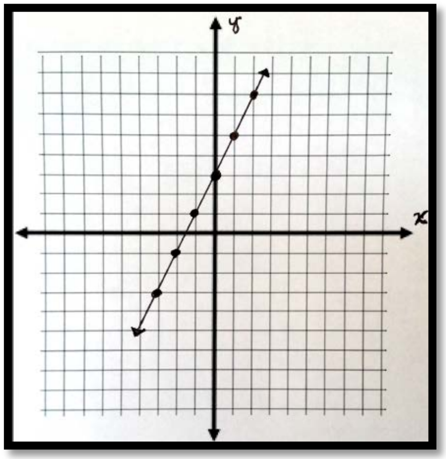
Question:
The sets of points below represent linear functions. Identify the slope of each set of points.
a. (9, 6) and (5, 2) b. (4, 1) and (-9, 1) c. (-3, 6) and (-3, 9) d. (8, 3) and (6, 7)

Solutions:
a. *m = 1* Slope is positive, increasing
b. *m = 0* Slope is zero, horizontal line
c. *m = Ø* Slope is undefined, vertical line
d. *m = -2* Slope is negative, decreasing

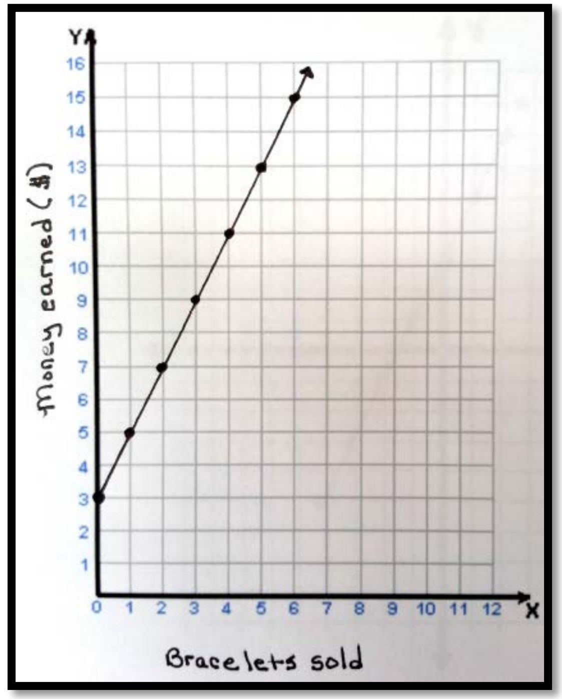
Students will pair, and the teacher will distribute ***"Pass" the Slope Activity*** (Attachment #9). The activity starts with each student completing the first questions. Partners will then switch papers. Partners will check the work done for the first question and complete the second question. Partners then switch papers again. This pattern continues until all the problems are completed by both partners. Teacher will circulate the classroom to ensure that both partners are contributing to the activity and to answer any student questions. Teacher will close the activity by reviewing any misconceptions and asking wrap up questions. One question might be, "Did you find a pattern (MP7) that could assist in easy determination of slopes that are zero and undefined?"

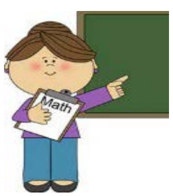
3. **Activity #3: F-IF.5**
Restricted Domains, Teacher Guided Instruction, Student Pair and Share, Student Independent Work (40 minutes estimated)

All linear functions have domains that included all real numbers. For example, *f(x) = 2x + 3* does not have any restrictions on the input or *x* values; positive, negative, or zero. The graph of the function is shown below.



Functions, especially linear functions, have many real world applications and can be given real world contexts. The function *f(x) = 2x + 3* can represent the amount of money Lenora makes selling bracelets. The *x* axis can represent the number of bracelets sold and the *y* axis can represent the amount of money earned. The original domain of all real numbers has to be restricted to positive integers when the function is given a real world context. Lenora cannot sell a negative number of bracelets. Most real world contexts place the graph of the function in the first quadrant where the *x* values (domain) is positive. Think about all the real world situations that involve positive domains: dimensions of an object (length, width, and height), time, distance traveled, items sold, cost, money, etc. The graph of the function *f(x) = 2x + 3* with a restricted domain representing the amount of money Lenora makes selling bracelets is shown below.





Describe a real world context for the function *f(x) = -12x + 72,* and state any restrictions on the domain. Describe the slope in context of the situation.

Solutions:

The math club made *72* t-shirts to give away at Rebel Pride Day. They plan to give away *12* t-shirts every hour during the event. The domain (x values) is time in hours and has to be positive. Rebel Pride Day cannot take place over negative hours. The slope is *-12* and models that fact that *12* t-shirts will be given away every hour. This represents a reduction of *12* t-shirts from the original *72* t-shirts each hour of the Rebel Pride Day.

Students will pair, and the teacher will distribute ***Model the Function*** (Attachment #10). Teacher will review directions with students, reminding students that problem #3 will be completed independently for homework. Student pairs will graph the original function and state the domain. Student pairs will create a real world situation for the original function. The function in context will be graphed with any domain restrictions. All graphs will be labeled correctly. The slope in context will be described. Teacher will circulate the classroom answering students' questions and guiding student work. Teacher will review any student misunderstandings by asking probing and/or thought provoking questions.

Day 4

1. **Activity #1:** ***Model the Function***
   Problem #3, Student Independent Homework (15 minutes estimated)

   Teacher will ask two or three students to present to the whole class their real world situation from homework problem #3. Teacher will review the domain restrictions suggested by the students and the labels for the axes. Teacher will review any final misunderstandings by asking probing and/or thought provoking questions.

2. **Activity #2: Review**
   Functions Lesson Plan Review - Teacher Guided Instruction (40 minutes if needed)
   Teacher will use the rest of this day as a review day as needed for students. If students need additional time for procedural skill and fluency, this day can be used for independent student work or pair and share as appropriate.

3. **Activity #3: Performance Based Assessment Task - Individual** (35 minutes estimated)
Teacher will review with the students one last time before giving the Performance based assessment task. This review will be at the discretion of the teacher and will cover items/topics/content specific to the students. Teacher will ask questions as needed to determine student understanding. Upon completion of the review, the teacher will distribute the Performance based assessment task.

Lesson Closure

1. Review the vocabulary words essential to the unit: relation, function, equation, input, output, *x* values, *y* values, domain, range, independent, dependent, *f(x)*, restriction, and slope.

2. Review the Bubble Map (5 Ways to Represent Functions).

3. Review the use of function notation such as *f(5)* and *f(x) + g(x).*

4. Review modeling functions and restricting the domain.

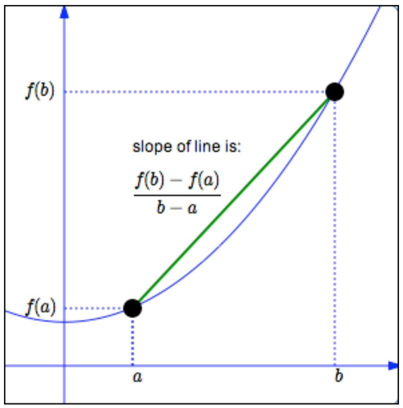
Essential Questions:

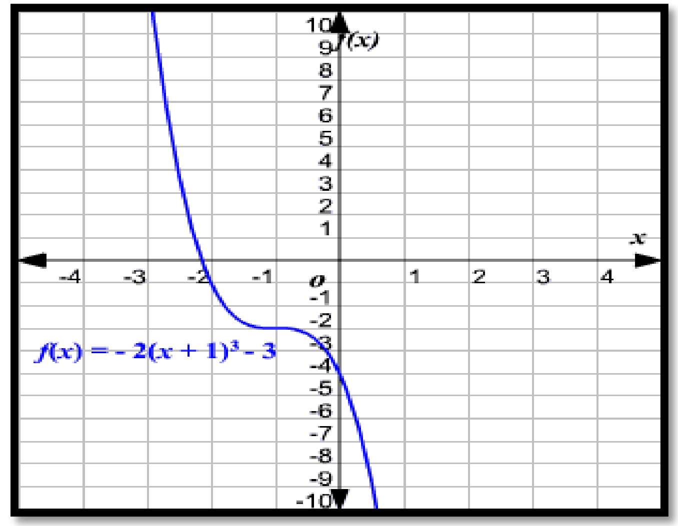
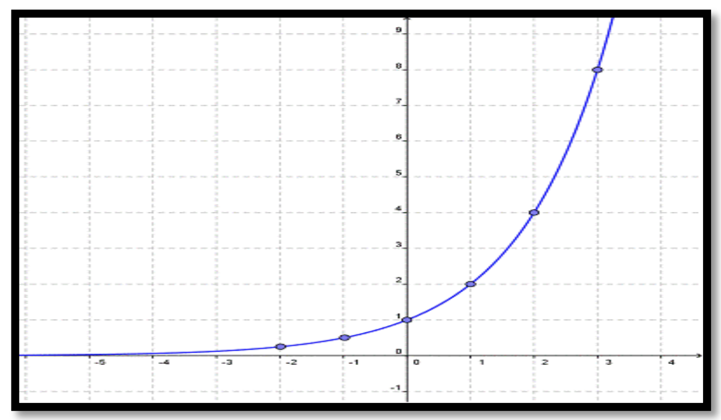
- What characterizes relations? Functions? Equations?
- What are the different ways to determine if a relation is a function?
- What makes a function different and/or better than a relation?
- When do domains have to be restricted for functions?
- How do you restrict domains?
- What is the domain for linear functions and why?
- What does it mean for a function to have a restriction?
- How do you find the slope of a function and what does the slope mean in context of a real world situation?

Standards for Mathematical Practice

✓ Make sense of problems and persevere in solving them.
✓ Reason abstractly and quantitatively.
✓ Construct viable arguments and critique the reasoning of others.
✓ Model with mathematics.
✓ Use appropriate tools strategically.
✓ Attend to precision.
✓ Look for and make use of structure.
✓ Look for and express regularity in repeated reasoning.

Supplemental Activities

Intervention	Enrichment
• **8.EE.6:** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. Use *x-y* coordinate geoboards to graph the functions below. The large bands can be used to represent the function and the small bands can be used to make similar triangles. This exercise will reinforce slope between two points, the slope formula, similar triangles, the y-intercept, and creating function/equations from a graph. If *x-y* coordinate geoboards are not available, this activity can use mini-white boards with *x-y* coordinate planes. Students would draw the similar triangles with expo markers. $f(x) = 2x - 5$ $f(x) = -x + 1$ • **8.EE.7b:** Solve linear equations in one variable. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. Use algebra tiles to reinforce expanding expressions using the distributive property. Students may not see that	• **F-IF.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* Calculate the average rate of change for a non-linear function from a graph by finding the slope of the line drawn between the points a and b. This line is called a secant line.  The secant line will be studied in Algebra III and Calculus when learning about the difference quotient and limits. For each non-linear function graphed below, draw and label the secant line between the two identified points and determine the slope of the secant line. 1.

f(x) = 10x – 8 and *f(x) = 2(x – 6) + 4(2x + 1)* are equivalent. Algebra tiles can show struggling learners the visual representation of both functions and how both functions are equivalent. Use algebra tiles to show that the following sets of functions are equivalent. a. *f(x) = 9x + 10* and *f(x) = 4(x + 5) + 5(x – 2)* b. *f(x) = -2x + 16* and *f(x) = 2(3x + 2) + 4(-2x + 3)*	 2. 

Performance Based Assessment Task	
Math Task	**Rubric/ Plausible Student Response(s)**
Create and graph a linear function with a real world context starting from a set of ordered pairs.	
a. Write a relation containing six ordered pairs and prove that the relation is a function.	a. **Write a relation containing six ordered pairs and prove that the relation is a function.** Relation of six ordered pairs written in set notation **10 pts** Justification that relation is function **10 pts**
b. Graph the ordered pairs and write the linear function.	b. **Graph the ordered pairs and write the linear function.** Graph of six ordered pairs with axes labeled **10 pts** Use of graph to find slope and y-intercept **10 pts** Linear equation written in function notation **10 pts**
c. Create a real world situation for the linear function.	c. **Create a real world situation for the linear function.** Real world situation explained with details and reasoning **20 pts**
d. Graph the linear function in context, restricting the domain and labeling the axes.	d. **Graph the linear function in context, restricting the domain and labeling the axes.** Graph of linear function in context with axes labeled **20 pts** Explanation of domain restriction **10 pts** **Total 100 pts**

MISSISSIPPI DEPARTMENT OF EDUCATION

Ensuring a bright future for every child

Lesson Plan Attachments

Attachment #1

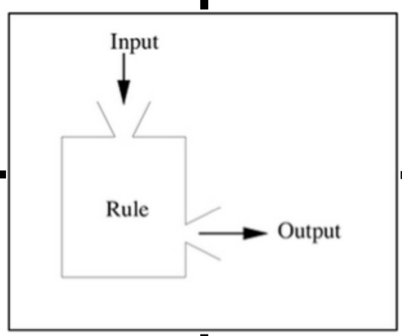
Mississippi Department of Education
Ensuring a bright future for every child

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Frayer Model – Input Output Model

Identify/Name

Facts/Characteristics



Example

Non-Example

Relation Vocabulary 4-Part Foldable

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

WORD	DEFINITION	PICTURE/SYMBOL	EXAMPLE/NONEXAMPLE

Attachment #3

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Directions: Cut along the outside border. Fold the center line. Laminate if desired.

Dependent

Independent

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Directions: Respond to the items by displaying the appropriate term from Attachment #3.

1. A diver tries to find the opening to a cave in Lake Hatter. The diver cannot make the dive if the water temperature is below 40°F. The temperature of the water was measured at different depths of the lake.

What is "the temperature"? Dependent or Independent?

What are "the depths"? Dependent or independent?

2. **Your class was challenged to compete in the Academic Bowl for your district. The week before the competition you studied an hour longer each night. Your class scored in the top three for your district.**

What is "the time spent studying"? Dependent or Independent?

What is "the team score"? Dependent or Independent?

3. **You drive at a rate of 60 mph for *t* hours.**

What is "the hours you drive"? Dependent or Independent?

What is "40 mph"? Dependent or independent?

What is "the distance traveled"? Dependent or Independent?

4. **When exercising, you burn 20 calories a minute.**

What is "calories burned per minute'? Dependent or Independent?

What is "number of minutes exercised'? Dependent or Independent?

What is "number of calories burned"? Dependent or Independent?

5. **Michael gets paid for the mushrooms he harvests in the mountains by weight. He collects $1.50 per pound for the mushrooms.**

What is "the weight of mushrooms harvested"? Dependent of Independent?

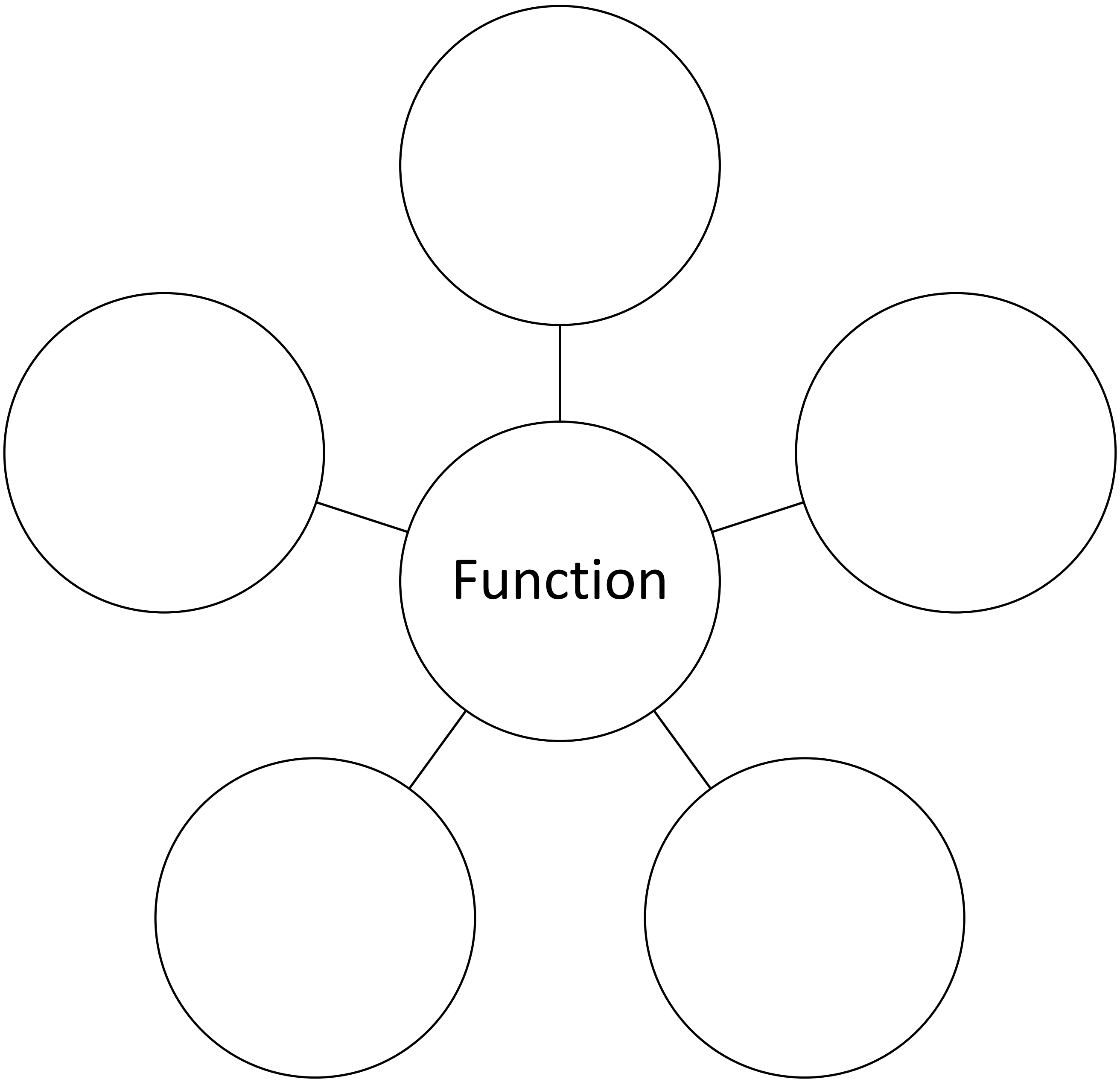
What is "$1.50 per pound"? Dependent or Independent?

What is "the amount Michael is paid"? Dependent or Independent?

Attachment #5

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Bubble Map Template



Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Ways to Represent Functions Poster Project

Materials Needed:

Poster board
Markers
Colored pencils
Graph paper
Ruler
Glue stick

Student Pair Directions:

a. Write two sets of ordered pairs, labeling them Set A and Set B. Each set needs to contain six ordered pairs. One set will be a relation and one set will be a function.

b. Choose three of the five "Ways to Represent Functions" to be displayed on your poster board. One of the three must be "Graphs of Functions".

c. Use each of the three ways to identify the two sets as either a relation or a function. Explain or justify the reasoning behind the identification. List the domain and range for the set identified as a function.

d. Disiplay all the information neatly on a poster board. Guidelines for displaying work can be found in the rubric on page 2.

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Scoring Rubric

Creating Sets of Ordered Pairs	
Two sets of ordered pairs created	**___/5 pts**
Ordered pairs labeled, Set A and Set B	**___/5 pts**
Each set contains six ordered pairs	**___/5 pts**
Choosing the Processes	
3 "Ways to Represent a Function" (one being Graphs of a Function)	**___/4 pts**
Labeling for Way #1 on the Poster Board	
Way #1 Titled	**___/2 pts**
Work clearly shown	**___/10 pts**
Sets of ordered pairs identified as relation or function	**___/4 pts**
Justification/Reasoning	**___/4 pts**
Domain written as set and labeled	**___/4 pts**
Labeling for Way #2 on the Poster Board	
Way #2 Titled	**___/2 pts**
Work clearly shown	**___/10 pts**
Sets of ordered pairs identified as relation or function	**___/4 pts**
Justification/Reasoning	**___/4 pts**
Domain written as set and labeled	**___/4 pts**
Labeling Graphs of Functions on the Poster Board	
Graphs of Functions Titled	**___/2 pts**
Graph of both sets of ordered pairs on poster board and labeled	**___/4 pts**
Vertical Line Test identified on graphs	**___/6 pts**
Sets of ordered pairs/graphs identified as relation or function	**___/4 pts**
Justification/Reasoning	**___/4 pts**
Domain written as set and labeled	**___/4 pts**
Poster	
Creative title (at top)	**___/5 pts**
Names of students (on back)	**___/4 pts**

Total Points Earned___________________/100 pts

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Function Notation Review

Directions:

Part A. Teacher may choose from the following items and ask students to rewrite the equation using function notation. All items listed below have been evaluated and determined to be functions.

$y = -2x - 3$	$y = 7x + 4$	$y = 3x - 8$
$y = \frac{1}{2}x + \frac{3}{4}$	$y = -5x + 2$	$y = 9x + 11$
$y = x - \frac{1}{4}$	$y = -17x + 6$	$y = 4x - 15$

Part B. Teacher may choose from the following functions and instruct students to evaluate the functions for each value given.

1. $f(x) = 6x + 3$ — $f(-1)$, $f(2)$, and $f(0)$
2. $f(x) = \frac{1}{2}x - 4$ — $f(8)$, $f(6)$, and $f(4)$
3. $f(x) = -3x + 9$ — $f(-5)$, $f(0)$, and $f(3)$
4. $f(x) = 0.75x + 12$ — $f(12)$, $f(16)$, and $f(24)$

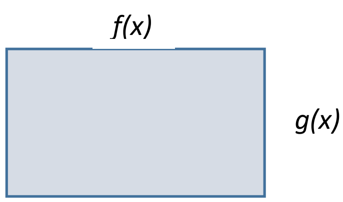
Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Function Notation Activity Packet

Directions:

Work with your partner to complete each section of this activity packet. Complete each section as specified in each step.

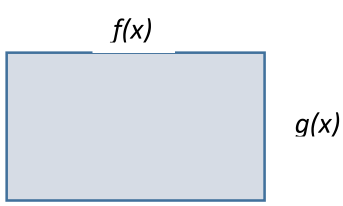
Part 1A. Use the diagram below to answer the following questions, $f(x) = 3x + 2$ and $g(x) = 5$



1. Determine the area and perimeter for the shape? Write your response as an algebraic expression.

2. If the perimeter is *68* units, what is the value of *x*?

Part 2A. Use the diagram below to answer the following questions, $f(x) = 2x – 6$ and $g(x) = 3$



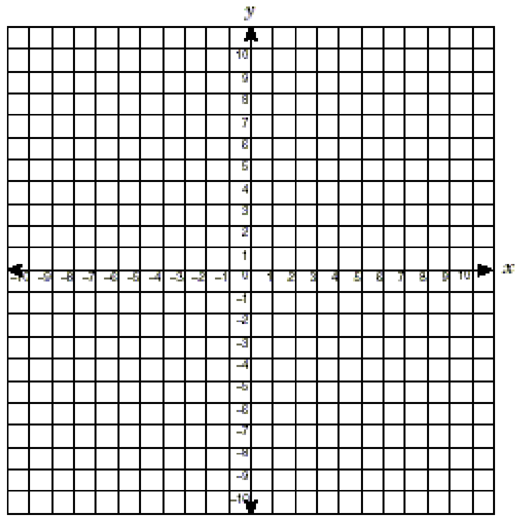
1. Determine the area and the perimeter for the shape? Write your response as an algebraic expression.

2. If the area is *72* units sq, what is the value of *x*?

Attachment #8

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Part 1B. For the function *f(x)*, rewrite the following items as a coordinate point and graph them on the given coordinate plane below. Explain how the graph proves that the given values represent a function.



f(-9) = -3

f(-3) = 1

f(0) = 2

f(1) = 1

f(4) = 8

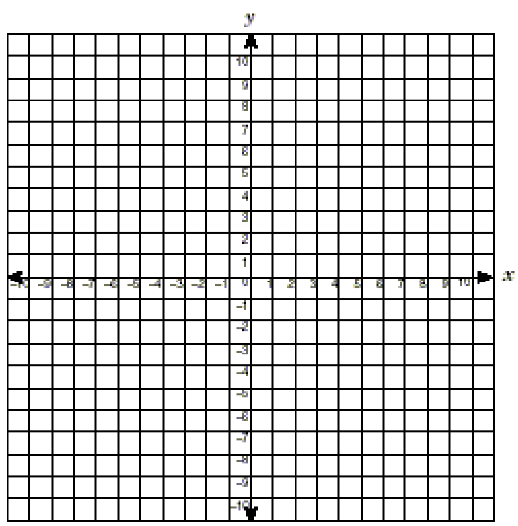
f(7) = -8

f(8) = 0

____

____

Part 2B. For the function *f(x)*, rewrite the following items as a coordinate point and graph them on the given coordinate plane below. Explain how the graph proves that the given values represent a function.



g(-4) = 7

g(-1) = -6

g(0) = 0

g(2) = 8

g(3) = -9

g(5) = 2

g(8) = 7

Part C. Use the following function rules to evaluate the given expressions.

$f(x) = x^2 + 3x + 2$ $g(x) = 9$ $h(x) = -9x + 12$ $k(x) = 2x + 6$

1. $f(-2)$

2. $g(0)$

3. $h(5)$

4. $k(-3)$

5. $f(x) + g(x)$

6. $h(x) + g(x)$

7. $h(x) + k(x)$

8. $k(x) - h(x)$

9. $k(x) * g(x)$

10. $f(x) * g(x)$

Part D. Create two input/output tables, one for a relation and one for a function. Justify the values you have placed in each table. Write the function rule for the table that represents a function.

Relation

Input	Output

____
____
____
____
____
____
____

Function

Input	Output

____
____
____
____
____
____

Attachment #9

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Pass the Slope Activity

Directions:

Work with a partner to calculate and describe the slope for each pair of points listed below. Use the slope (rate of change) formula to complete all the questions in this activity.

Individually, complete the first question and then switch papers with your partner. Check your partner's work and then complete the second question. Switch papers again. Continue this process until all questions have been answered.

a. *(4, 6)* and *(-9, 12)*

b. *(12, 3)* and *(12, -8)*

c. *(5, 7)* and *(-1, 0)*

d. *(6, 2)* and *(10, 8)*

e. *(4, 3)* and *(-2, 3)*

f. *(11, 9)* and *(-13, 15)*

g. *(7, 1)* and *(-2, 1)*

h. *(-2, 8)* and *(4, 12)*

i. *(-11, 4)* and *(-11, 10)*

j. *(6, -1)* and *(8, -2)*

Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Model the Function

Directions:

Work with your partner to answer Problems 1 and 2. Problem 3 will be done independently for homework.

For each function,

a. Graph the original function and state the domain.

b. State a real world context for the function.

c. State any domain restrictions on the function in context.

d. Graph the function in context and label the axes correctly.

e. Explain the slope of the function in context.

1. $f(x) = 150x + 1500$

2. $f(x) = 0.75x + 75.00$

3. $f(x) = -25x + 750$