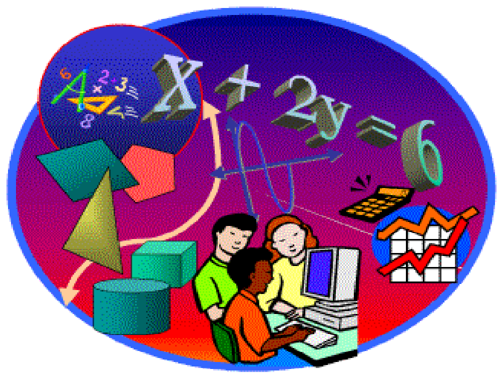
College- and Career-Readiness Standards for Mathematics



Exemplar Lesson Plan

"Polynomials- Maximum or Minimum?"

A-SSE.2

A-APR.3

F-IF.4

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Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

COURSE: Algebra II	
Title: *Polynomials, Maximum or Minimum?* **Estimated Duration: 3 to 4 Days**	**Real World Purpose:** Polynomials can be found in many different and diverse industries such as aerospace, architecture, financial planning, legal, and construction to name a few. But what about polynomial division? Ever think you might do that again after a high school or college math course? You will if you become a pharmacist and need to determine the correct amount of a drug to administer to a patient. The same goes for a chemist. They use polynomial division to derive chemical formulas.

I Can:

A-SSE.2: Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

A-APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

F-IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

Prerequisite Skills:

A-SSE.1: Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret *P(1+r)*n as the product of *P* and a factor not depending on *P*.

A-APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A-APR.2: Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

A-APR.6: Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-REI.4: Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a ± bi$ for real numbers a and b.

A-REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Materials/Resources:	**Key Vocabulary:**
• www.classzone.com • www.msemac.redwoods.edu/~kyokoyama/math106/Math380pdf • www.illustrativemathematic.org • www.engageny.com • https://mathstackexchange.com • https://wvde.state.wv.us/strategybank/FrayerModel.html • Algebra tiles • Graphing calculator • Graph paper • Mini-white board • Expo marker • Eraser • Algebra II Pearson Prentice Hall • **Attachments (Total: 8)**	• Quadratic • Linear factor • Long division • Synthetic division • Polynomial • Zero • Solution • Root • Relative maximum • Relative minimum • Increasing interval • T-chart • Punnett square • Sum of Cubes • Difference of Cubes • Difference of Two Square • Complex solutions • u-substitution • X-intercept • Decreasing interval • Union • Multiplicity • Zero Product Property

Lesson Introduction

Student Exploration Activity: What does the graph model?
Student Independent Work (15 minutes estimated)

Directions:
The teacher will distribute the activity ***Real-World Context – Graph – Function*** (*Attachment #1*). The teacher will review the directions at the top of the activity. The teacher will give students 10 minutes to match the real-world situation to the graph and to match the graph to the function without the use of a calculator. The teacher will spend 5 minutes as closure reviewing students' work sample and introducing the unit *Polynomials, Maximum or Minimum?*

Activity Objective:
Using prior knowledge, students will work independently to determine which real-world situation models the polynomial represented by each graph. Students will have an opportunity to match the graph of the polynomial to the function. The teacher will explain the general characteristics of a polynomial function and how to relate the graph to its function.

Lesson Activities

Guidelines for Group Work

- The teacher will remind students during the Pair and Share activities that collaboration is required. Students will share work collaboratively in solving the task given. Once one student finds the solution, the other student needs to check their work. Students will take turns problem solving until the task is complete. Students will use the "Flag It" technique to note any areas of difficulty during the activity on sticky notes.
- The teacher will remind students during the Group activities that collaboration is required. Students will need to take turns completing parts of the task until the task is complete. When a student is not identifying or determining solutions, he/she should be checking the work of another student or asking questions of the group. Students will use the "Flag It" technique to note any areas of difficulty during the activity on sticky notes.
- The teacher will circulate throughout the classroom to monitor student work and to guide them through the activity using any form of questioning technique they deem appropriate. The teacher is encouraged to ask guiding questions that will encourage productive struggle and critical thinking skills. The teacher will also monitor individual student understanding and general misconceptions.

Day 1

1. **Activity #1: A-SSE.3a**
   Mini-Review Factoring Quadratic Expressions - All students using Mini-white Boards and Algebra Tiles (15 minutes estimated)
   The teacher will review factoring quadratics using mini-white boards which allow the teacher to assess students' prior knowledge. The teacher will allow students to use Algebra Tiles if needed for struggling learners. Quadratics may range from an easier type $x^2 + 4x$ (where the leading coefficient is $a = 1$) to a harder type $4x^2 - 4x - 15$ (where the leading coefficient is $a \neq 1$). The teacher will select 5 to 10 quadratics to review during this activity. The teacher will remind students that they can also use T-charts and Punnett Squares if

needed to factor the quadratics. The teacher will guide students through the activity using effective questioning techniques. Two questions a teacher might ask include, “Do you remember any factoring steps from Algebra I that could assist you?” and “What are the factors of 4 and 15?” The teacher will allow for opportunities at the beginning of the review for students to have productive struggle with factoring techniques for quadratics. Students will also have any opportunity to assess their own level of understanding by noting issues that arise during instruction.

Activity Objective:
Students were introduced to factoring quadratics in Algebra I and furthered their skills in factoring quadratics and finding zeros in this course. Students will review their prior knowledge and fluency of factoring quadratics to better prepare for Day 2 Activity #2.

2. **Activity #2: A-APR.6**
   Mini-Review Long Division of Polynomials - Students using white board and paper/pencil at desk (20 minutes estimated)
   The teacher will call two students to the board to divide a 3rd degree or higher polynomial by a given linear factor. For example, $x^3 - 13x + 12$ divided by $x + 4$. One student will perform long division and another student may perform synthetic division. While at their desk, the remainder of the class may use paper/pencil and choose any method they prefer. The two students at the board will compare their solutions. Students at their desks will also check their work. The teacher will remind students that division is a technique to find factors of polynomials. The teacher will guide students to factor the quadratic into linear factors as well. The teacher will choose several different types of long division problems to showcase during the review activity. The teacher will call upon different pairs of students for each problem as time permits. The teacher will ask guided questions during the review activity to assess students’ conceptual understanding and circulate throughout the classroom to assess students’ procedural skill and determine fluency.

   Activity Objective:
   Students will review long division of polynomials as a procedural skill while improving their fluency with factoring. Students will have an opportunity to assess their understanding of polynomial long division during the review. Students will recall the significance of a remainder with completing tasks dealing with long division or synthetic division. Students will be able to factor a polynomial into linear factors for certain cases.

3. **Activity #3: A-SSE.2**
Factoring Polynomials using Structure - Teacher Guided Instruction (30 minutes estimated) **and Pair and Share** (30 minutes estimated)
The teacher will write the following phrases on the board: *Factor by Grouping, Pattern for Sum and Difference of Cubes, Difference of Squares,* and *Factor using a Quadratic Pattern*. The teacher will give students 5 minutes to brainstorm about their prior knowledge of quadratics that will be transferable to their work with polynomials. The teacher will explain to students that they will learn how to factor polynomial expressions using structure as a way to rewrite the expression into a form that is easier to factor. The teacher will inform students that the goal is to use different techniques to continue factoring until all linear factors are made.

The teacher will instruct students on the following techniques for factoring some 3rd degree polynomials:

Ex: Factor by Grouping
$x^3 + 4x^2 - 16x - 64$

Option 1	Option 2
$(x^3 + 4x^2) - (16x + 64)$	$(x^3 - 16x) + (4x^2 - 64)$
$x^2(x + 4) - 16(x + 4)$	$x(x^2 - 16) + 4(x^2 - 16)$
$(x^2 - 16)(x + 4)$	$(x + 4)(x^2 - 16)$
$(x + 4)(x - 4)(x + 4)$	$(x + 4)(x + 4)(x - 4)$

The teacher will explain that both options are correct. The key is to group correctly so that when the GCFs are found, the factors will match (see highlighted factors above). Students will also use the Difference of Two Squares to factor completely. The teacher will assess students' conceptual understanding by asking if there is any other possible grouping for this polynomial.

Ex: Pattern for Sum and Difference of Cubes

$(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$ $\quad$ $(x^3 + 27) = (x + 3)(x^2 - 3x + 9)$

$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$ $\quad$ $(x^3 - 125) = (x - 5)(x^2 + 5x + 25)$

The teacher will review the above pattern and use double distribution to prove the pattern works for these special cases. The teacher will show students the difference between the *Difference of Cubes* pattern and the *Difference of Two Squares* pattern. The teacher will pose the question, "Is there a Sum of Two Squares pattern?" The teacher will allow students a few minutes to reach a decision. The teacher will allow 2 to 3 students explain their thinking on the board. The teacher will remind students that the quadratic formed from the Sum and Difference of Cubes factors into two complex solutions.

The teacher will instruct students on the following techniques for factoring some 4th degree polynomials:

Ex: Difference of Two Squares

$x^4 - 81 = (x^2)^2 - (3^2)^2$
$= (x^2 - 3^2)(x^2 + 3^2)$
$= (x - 3)(x + 3)(x^2 + 9)$

$16x^4 - 256 = (4x^2)^2 - (4^2)^2$
$= (4x^2 - 4^2)(4x^2 + 4^2)$
$= (2x - 4)(2x + 4)(4x^2 + 16)$

The teacher will explain how the Difference of Two Squares factoring technique can be applied to a polynomial expression. This technique will be repeated twice. The teacher will show students that one of the first factors creates an additional Difference of Two Squares (see highlighted factors above) and the quadratic that cannot be factored has two complex solutions.

Ex: Factor using a Quadratic Pattern

u-substitution

$x^4 - 12x^2 + 32 = (x^2)^2 - 12(x^2) + 32$
$= u^2 - 12u + 32$
$= (u - 4)(u - 8)$
$= (x^2 - 4)(x^2 - 8)$
$= (x - 2)(x + 2)(x^2 - 8)$

The teacher will explain that a quadratic expression can be made by rewriting x^4 as $(x^2)^2$ and substituting u for x^2. Students can then factor using techniques for simple quadratic expressions learned in Algebra I or earlier in Algebra II. After the quadratic expression is factored, the teacher will show students how to rewrite the quadratic expression replacing the u with x^2. Now students will factor $(x^2 – 4)$ as the Difference of Two Squares but $(x^2 – 8)$ does not factor by any of these techniques. The teacher will explain that the goal of factoring is to be able to obtain the zeros of the polynomial from linear factors.

The teacher will check for students' conceptual understanding by completing ***Special Cases Review*** (*Attachment #2*). The teacher will group students in pairs and instruct them to complete the template with as many facts as possible about each special case. The teacher will review the directions at the top of the activity and explain the objective of the activity. The teacher will allow 15 minutes for this activity and will review all responses by asking students to present and justify their responses under a particular column on the template. Students will be allowed to use ***Special Cases Review*** (*Attachment #2) a*s notes for the next activity.

Students will remain with their partner as the teacher distributes ***Factoring Polynomial Expressions*** (*Attachment #3*). The teacher will review the directions at the top of the activity and explain the objective. Students will use factoring by special cases, factoring techniques for quadratics, and long division to find the linear factors for the polynomial expressions. At the end of the activity, the teacher will review some of the "Flag Its" and student misconceptions.

Activity Objective:
Students will have an opportunity to become more fluent with factoring simple quadratic expressions by seeing structure and rewriting polynomial expressions into a form that is easier to factor. The teacher has had several opportunities to assess students' understanding and students have had opportunities to assess themselves. Students will now be able to use the linear factors or quadratics to find the zeros of the polynomial functions tomorrow and sketch a graph.

Day 2

1. **Activity #1: Opening - Word Wall** (10 minutes estimated)
   The teacher will review the following key vocabulary terms from the word wall for today's lesson on standard A-APR.3**.**

Zero – A zero of a function is any value of the variable for which the function is *0*. On the graph of a function, each *x*-intercept represents a zero. A zero may be real or complex.

Root – A root of a function is the input value for which the value of the function is zero. A root of an equation is a value that makes the equation true. The words root and zero may be used interchangeably.

Solution – A solution of an equation is a number that makes the equation true.

X-intercept – The point at which a line crosses the x-axis (or the *x*-coordinate of that point) is an *x*-intercept.

Students should remember the following statement*: The zero(s) of a polynomial function f is (are) a solution(s) to the equation $f(x) = 0$. If $(x – a)$ is a factor of a polynomial function, then $f(a) = 0$ and a is a zero of f*. The teacher will describe how zeros and roots can be found using the Zero Product Property which states that linear factors are set equal to *0* and solved for *x*. Solutions to $f(x) = 0$ can be found first by factoring and then using the quadratic formula, completing the square, or taking a square root. Solutions can be real or complex. Solutions can be found from a graph of the function, also known as the *x*-intercept. The teacher will remind students that *x*-intercepts occur at $(x, 0)$ and are easily located on the graph of the function.

<u>Transition Activity - Pair and Share and Student Presentation</u> (15 minutes estimated)
The teacher will pose the following questions, "Does the number of solutions have to be the same as the number of zeros? Why or Why not?" The teacher will write the following factored polynomial equation on the board.

$(x – 3)(x – 3)(x + 2)(x + 2)(x + 1)(x – 4) = 0$

Identify:
Zeros
Solutions to $f(x) = 0$

Students will work with a close partner to come to a consensus on an answer to the question and write a justification for their answer. The teacher will ask 2 to 3 pairs of students to present their findings and justification to the class. The teacher will make sure that students understand that for this particular problem the number of zeros and the number of solution are not the same. The zero *"3"* has a multiplicity of *2*, which means it occurs twice. The solution *"3"* only occurs once. The same as for the zero *"-2"*. The answer to the problem on the board is

$(x - 3)(x - 3)(x + 2)(x + 2)(x + 1)(x - 4) = 0$

Zeros: *3, 3, -2, -2, -1, 4*
Solutions: *3, -2, -1, 4*

Activity Objective:
Students will be able to recognize key vocabulary terms in activities, real-world problems, and performance based assessment tasks. Students have prior knowledge to factor polynomials into linear and quadratic factors and students are able to use the Zero Product Property to find the zeros or roots of the polynomials. Students understand that not all polynomials have the same number of zeros as solutions due to multiplicity.

2. **Activity #2: A-APR.3**
Identify Zeros - Teacher Guided Instruction and Pair and Share (20 minutes estimated)
The teacher will explain that for this activity students will combine their conceptual understanding and procedural skill of factoring to find the zeros. Students can use any factoring technique for quadratics from Algebra I and Algebra II, factoring techniques for special cases, or long or synthetic division for polynomials to factor the polynomial functions. Students will also use the Zero Product Property to find the zeros of the polynomial functions. Students will stay with their same partners for this activity. The teacher will distribute the activity ***Factor and Find the Zeros*** (*Attachment #4*). The teacher will review the directions at the top of the activity and explain the objective of the activity. The teacher will explain the enrichment task included at the end of the activity. The enrichment task will require students to apply current and prior knowledge to find all the solutions - real and complex - for a fifth degree polynomial. At the end of the activity, the teacher will review some of the "Flag Its" and student misconceptions.

Activity Objective:
Students will be able to identify zeros from the factored form of a polynomial equation. Students will be able to create the factored form of polynomial equations using a variety of factorization techniques acquired from Algebra I and this course. Students will have an opportunity to explore an enrichment task to find all solutions - real and complex - for a fifth degree polynomial.

3. **Activity #3: A-APR.3**
Construct a Rough Graph - Teacher Guided Instruction, Student Presentation, and Student Independent Work (50 minutes estimated)
Students will begin by playing ***Where's my Zeros, Where's my Graph?*** (*Attachment #5*). Each student will receive a different colored card: orange (function cards), blue (graph cards), or green (zero cards). When the teacher signals to begin, students will circulate through the classroom to find their other two matches. Once a group of three has been made, the group will discuss their strategy for finding their matches and be prepared to share with the class. Once all groups have been formed, the teacher will call upon *3* groups to share their strategies for success and ask if any groups would like to share their struggles and how they overcame them. The teacher will be able to assess student understanding through appropriate questioning as students report out.

Activity Objective:
Students will have an opportunity to see the graphs of polynomials in factored form and use MP. 7 and MP. 8 to help unravel the mysteries of the activity. Students that need additional scaffolding should experience success with this activity because the polynomials have been partially factored for them.

Students should now be able to recognize the zeros of a polynomial function on a graph. The teacher will demonstrate how to sketch a rough graph of a polynomial using zeros. The teacher will write the following equation with partial factorization on the board.

$f(x) = x^3 – 52x – 96$ with $(x + 2)$ as a linear factor

The teacher will begin solving the problem with long division. The final factored form of the polynomial is

$f(x) = (x – 8)(x + 6)(x + 2)$

Zeros: *-6, -2, 8*
Solutions: *-6, -2, 8*
x-intercepts: *-6, -2, 8*

The teacher will ask the question, "At which *x* values can the function change from being positive to negative or vice versa?" The students' response should at be the *x*-intercepts. The teacher will ask students to represent the *x*-intercepts as four intervals: $x < -6$, $-6 < x < -2$, $-2 < x < 8$, and $x > 8$. The teacher will ask students, "To Identify the critical aspects of each interval. Picture in your mind the graphs of polynomials from our previous activities." The students' response include the fact that the function will change sign at the *x*-intercepts and on each interval the graph will either be above or below the *x*-axis. The teacher will further ask "How is it possible to tell if the function is positive or negative on an interval?" The teacher ask guiding questions that build students' conceptual understanding. The teacher will assist students in seeing that by evaluating the function at a given point on the interval they can sketch a rough graph of the function.

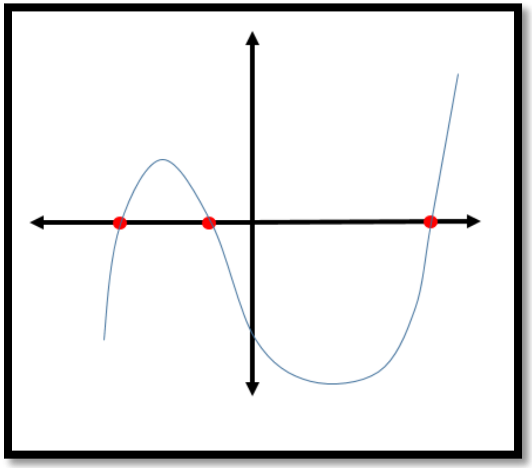
For the interval $x < -6$ find $f(-8) = -192$. Because the value is negative, the graph is below the *x*-axis.

For the interval $-6 < x < -2$ find $f(-4) = 48$. Because the value is positive, the graph is above the *x*-axis.

For the interval $-2 < x < 8$ find $f(2) = -192$. Because the value is negative, the graph is below the *x*-axis.

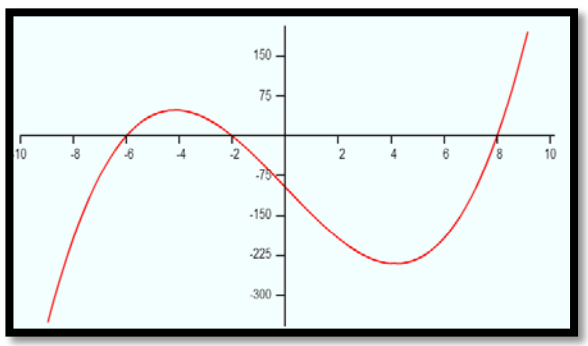
For the interval $x > 8$ find $f(11) = 663$. Because the value is positive, the graph is above the *x*-axis.

The teacher will ask students to create a rough sketch of the graph using the x-intercepts and the information listed above.



The teacher will then instruct students to graph the function using their graphing calculator using the following window settings:

WINDOW
X min = -10
X max = 10
X scl = 2
Y min = -350
Y max = 200
Y scl = 75



Students will compare their sketch to the graph generated by their calculator. The teacher will allow 5 minutes for general discussion and questions.

Students will revisit the activity ***Factor and Find the Zeros*** (*Attachment #4*). This time the activity will be completed independently. The teacher will instruct students to use the zeros found in the activity to construct a rough graph of the polynomials. Students will need to write intervals for the zeros and pick points on those intervals to determine whether the graph is above or below the graph at those points. After students construct their rough graphs, the teacher will display the actual graph on the smart board using a graphing program. Students can also use hand-held graphing calculators

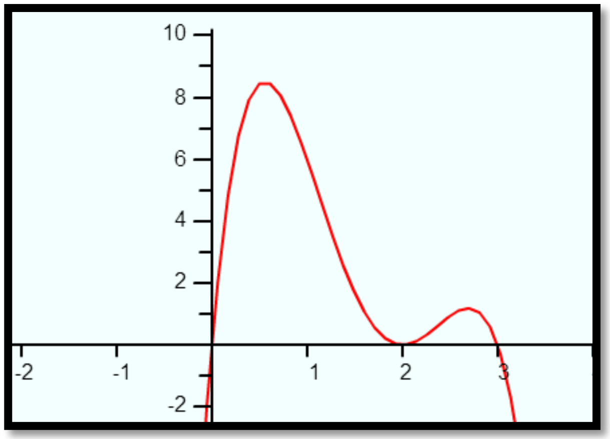
Activity Objective:
Students will use factoring and other skills to find zeros in order to sketch the graph of a polynomial function. Students will learn that functions change from positive to negative and negative to positive at x-intercepts where $f(x) = 0$.

Day 3

1. **Activity #1 F-IF.4: Opening** (15 minutes estimated)
   The teacher will have the following problem on the board when students come into the class room.

   The local 4-Flag's Amusement Park is designing a children's roller coaster and has tried different functions to model the height of the roller coaster during the first *300* yards. The chief designer came up with the following function to describe what he believes would make a fun start to the ride:

   $H(x) = -3x^4 + 21x^3 – 48x^2 + 36x$, where $H(x)$ is the height of the roller coaster in yards when the roller coaster is $100x$ yards from the beginning of the ride.



Students will answer the following questions:

a. Would the roller coaster be fun to ride based on the graphical model of the function? Justify your answer.

b. Can you determine any obvious *x*-values from *H(x)* where the roller coaster would be at height *0* ?

c. Can you identify from the graph when the roller coaster is *0* yards above the ground?

d. What do the *x*-values from part (c) represent in terms of the distance from the beginning of the ride?

e. Justify your answer in part (c) by factoring the polynomial *H(x)* knowing that *x* and *(x – 3)* are factors.

The teacher will randomly ask students to identify their responses to the above questions and will ask for agreement/disagreement from other students. The teacher will assess students' conceptual understanding and application through probing questions. Two questions a teacher might ask include, "What is wrong with the roller coaster model at a distance of *0* yards and *300* yards? Why might this not be the amusement park's initial design option?"

Activity Objective:
Students will have an opportunity to apply a real-world situation to a polynomial function by deconstructing the graph of the polynomial function. Students will take their memories of a childhood/teenage activity and apply mathematical concepts to the activity.

2. **Activity #2: F-IF.4**
   Recognizing Key Features of a Polynomial Graph - Teacher Guided Instruction and Pair and Share (60 minutes estimated)
   The teacher will add the following terms to students' word wall for today's lesson on the standard F-IF.4**.**

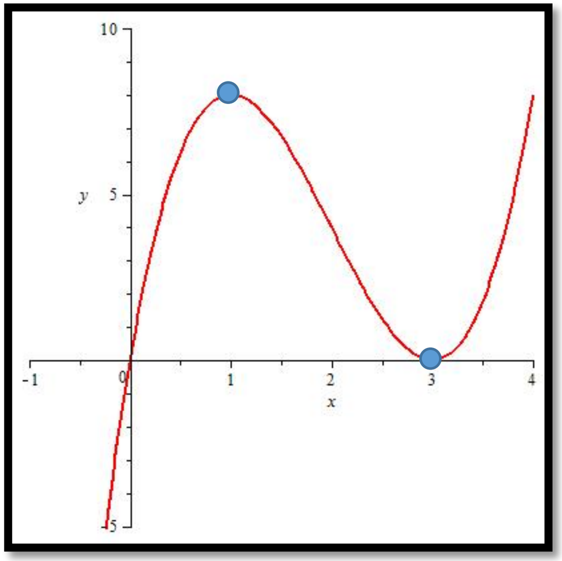
   Relative maximum – The *y*-value of a point on the graph of a function that is higher than the nearby points of the graph is a relative maximum.

   Relative minimum – The *y*-value of a point on the graph of a function that is lower than the nearby points of the graph is a relative minimum.

   Increasing interval – For a function $f(x)$ over an interval where $x_1 < x_2$, $f(x)$ is increasing if $f(x_1) < f(x_2)$.

   Decreasing interval – For a function $f(x)$ over an interval where $x_1 < x_2$, $f(x)$ is decreasing if $f(x_1) > f(x_2)$.

   The teacher will display the graph of a function of a polynomial and use the graph to explain the above terminology. For the function displayed below, the relative maximum can be found at (1, 8) and the relative minimum can be found at (3, 0). The function is increasing on the intervals $(-\infty, 1) \cup (3, \infty)$. The function is decreasing on the interval (1, 3).



The teacher knows there are other components to the standard F-IF.4 (symmetry, end behavior, and periodicity). However, all components have been covered to model the polynomial max/min problems that will complete this lesson plan. The teacher will relate the idea of relative maximum and minimum to model the following problems: find the maximum volume of a box, find the minimum production cost for a company, or find the minimum amount of material needed. The teacher will use the following activity, ***Open Top Box Challenge*** (*Attachment #6*), as a modeling example for the class. The teacher will explain that students will work in groups to create a box with an open top and then determine which group in the class, if any, created the box with the maximum volume using regression techniques.

Activity Objective:

Students will first have an opportunity to complete a non-graphical approach for solving a maximization problem with a polynomial function. Then, students will justify the class table and final maximum volume using quadratic and cubic regression techniques.

The teacher will now guide students on how to construct a polynomial function requiring maximization. The teacher will advise students to use the "Flag It" technique to identify any areas of concern during the teacher guided instruction. The teacher will propose the following situation:

Company ABC sells widgets at *$75* per widget. Company ABC has *800* customers willing to buy the widgets for that price. Company ABC is thinking of increasing the selling price of its widget. The company knows that for every *$10* increase in price, there will be *40* fewer customers willing to buy the widget. What should Company ABC sell its widget for to maximize its income?

The teacher will ask students to calculate the company's original income and the equation used.

Income = # of widgets sold * price of widgets
= *800 * 75*
= *$60,000*

The teacher will then ask students to determine the variable in the situation given. The teacher will give adequate wait time if students struggle to define the variable as the number of *$10* increases. The teacher will guide students to the new maximum income equation.

$I(x) = (800 - 40x)(75 + 10x)$

The teacher will direct students to use technology to find the maximum for $I(x)$. The maximum income that Company ABC can make is *$75,625*. The teacher will review any "Flag Its" and student misconceptions identified at this time.

The teacher will now guide students through a minimization problem that uses a polynomial function. The teacher will advise students to use the "Flag It" technique to identify any areas of concern during the teacher guided instruction. The teacher will propose the following situation:

Company XYZ sells widgets and needs to minimize their production costs. If Company XYZ produces too many widgets, the extra widgets that are not sold go into storage which adds to production costs. If Company XYZ produces too few widgets, the cost to produce each individual widget is too high. The accounting department has created the following cost equation, *C(x)*, to determine how many widgets, *x* (in thousands), XYZ company needs to produce each week to minimize production costs:

$C(x) = 0.06x^2 - 9.6x + 36{,}000$

The teacher will direct students to use technology to find the minimum for *C(x)*. The number of widgets needed to minimize production costs is *80,000*. The teacher will review any "Flag Its" and student misconceptions identified at this time.

The teacher will distribute the activity ***Modeling Polynomial Functions*** (*Attachment #7*). Students will work individually to complete the activity. Students will find the maximum and minimum of polynomial functions and use the characteristics of graphs to answer real-world questions

Activity Objective:
Students will explore the graphical characteristics of 2nd and 3rd degree polynomial functions. Students will be able to find the maximum and minimum values for real-world problems. Students will also use the characteristics of a graph to answer other real-world questions.

3. **Activity #4: Teacher Review** (15 minutes estimated)
The teacher will review the students' findings from the activity ***Modeling Polynomial Functions*** (*Attachment #7)*. The teacher will review some of the "Flag Its" and student misconceptions from the activity. The teacher will review key concepts from the activity to review students for the Performance Based Assessment Task.

Activity Objective:
Students will receive feedback on Activity #3 and review for the Performance Based Assessment Task.

Day 4

1. **Activity #1: Polynomial Functions Lesson Plan Review - Teacher Guided Instruction** (60 minutes if needed)
   The teacher will use this day as a review day as needed for students. If students need additional time for procedural skill and fluency, this day can be used for independent student work or pair and share as appropriate.

2. **Activity #2: Performance Based Assessment Task - Individual** (35 minutes estimated)
   The teacher will review with students one last time before giving the performance based assessment task. This review will be at the discretion of the teacher and will cover key topics uncovered in the lesson. The teacher will ask questions as needed to determine student understanding. Upon completion of the review, the teacher will distribute the performance based assessment task.

Lesson Closure	
1. Lesson Closure will occur either at the end of Day 3 as homework or Day 4 during wrap-up. 2. Students will individually create a Frayer Model for Polynomial Functions (*Attachment #8*). 3. The teacher will display student's Frayer Models in the classroom.	**Essential Questions:** • How are polynomial functions different from linear functions? • How can synthetic and long division be used to factor a polynomial and what is the significance of the remainder? • What special cases are useful during the factoring of quadratics? • What special cases are useful during the factoring of 3rd degree and higher polynomials? • How can the shape of a graph help determine intervals of increasing and decreasing?

	• How can the shape of a graph help determine maximums and minimums? • What types of real-world situations can be modeled using polynomial functions?

Standards for Mathematical Practice

✓ Make sense of problems and persevere in solving them.
✓ Reason abstractly and quantitatively.
✓ Construct viable arguments and critique the reasoning of others.
✓ Model with mathematics.
✓ Use appropriate tools strategically.
✓ Attend to precision.
✓ Look for and make use of structure.
✓ Look for and express regularity in repeated reasoning.

Supplemental Activities

Intervention	**Enrichment**
• Standard A-SSE.3a Scaffold instruction so that students are able to factor and find zeros by beginning with quadratics that are already factored and set equal to zero. For example, the teacher can give students factored quadratic equations and ask students to find the zeros using the zero product property. What are the zeros? $x^2 + 7x + 10 = 0$ $(x + 5)(x + 2) = 0$ Students should be able to find $x = -5$ and $x = -2$.	• **Multiplicity**: Multiplicity is the number of times a factor shows up in the polynomial. A zero can occur multiple times in a polynomial. For example, the polynomial $f(x) = x(x - 3)^2(x + 5)(x - 7)^3$ has seven zeros. The zeros are $x = 0$ $x = 3$ multiplicity 2 $x = -5$ $x = 7$ multiplicity 3

$x^2 - 12x + 35 = 0$ $(x - 5)(x - 7) = 0$ Students should be able to find $x = 5$ and $x = 7$. $3x^2 + 13x + 12 = 0$ $(3x + 4)(x + 3) = 0$ Students should be able to find $x = -4/3$ and $x = -3$. The teacher can model through the use of Algebra Tiles, T-squares, Punnett squares, and other techniques to scaffold instruction for students struggling with factoring quadratics. For example, the teacher can give the quadratic equations and ask student to first find the linear factors and then find the zeros. The teacher will require students to know the Zero Product Property. Identify the linear factors and then find the zeros, for the following quadratic equations: $x^2 + 10x + 21 = 0$ $(x + 3)(x + 7) = 0$ zeros at $x = -3$ and $x = -7$ $2x^2 - 10x + 8 = 0$ $(2x - 2)(x - 4) = 0$ zeros at $x = 1$ and $x = 4$ $3x^2 + 2x - 5 = 0$ $(3x + 5)(x - 1) = 0$ zeros at $x = -5/3$ and $x = 1$	• **Write a polynomial from factors**: Students will write polynomials from zeros using their multiplicity. For example, write a polynomial function that has the following zeros: $x = 0$ multiplicity 2 $x = 4$ multiplicity 2 $x = 5$ $x = 1$ $x = -2$ multiplicity 3 $f(x) = x^2(x - 4)^2(x - 5)(x - 1)(x + 2)^3$ • Find all the zeros (real and complex) from the factored form of a polynomial using the Zero Product Property. $f(x) = x^2(x - 9)^3(x^2 + 5)(x + 1)$ The zeros are $x = 0$ multiplicity 2 $x = 9$ multiplicity 3 $x = \pm i\sqrt{5}$ $x = -1$

Performance Based Assessment Task

The Storage Box

Part A
Janie needs to find a storage box that has a minimum storage of 700 in^3 for beads and buttons. Janie finds a box at a thrift store and can use the equation $V(x) = x^3 + x^2 - 12x$ to find the volume of the box.

a. If the box has $width = x - 3$, write the linear expressions for the box's height and length. Rewrite the V(x) equation.

b. If the box's width is *6* inches, what are the dimensions for the box's height and length?

c. Find the volume of Janie's box.

Rubric/Plausible Student Responses

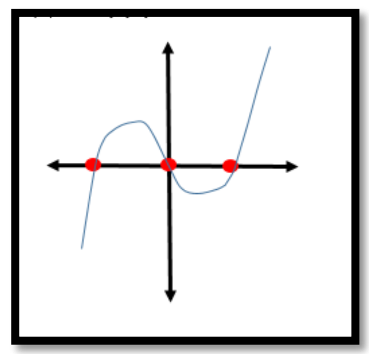
Part A
a.
$V(x) = x(x - 3)(x + 4)$ **(1pt)**
$width = x - 3$
$length = x + 4$ **(1pt)**
$height = x$ **(1pt)**

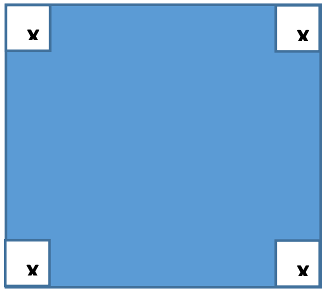
b.
$width = x - 3$
$6 = x - 3$
$9 = x$ and $height = 9$ **(1pt)**

$length = x + 4$
$length = 9 + 4$
$length = 13$ **(1pt)**

c.
$V = l*w*h$
$V = 13*6*9$
$V = 702\ in^3$ **(2pts)**

d. Yes.
The storage box will meet Janie's minimum storage requirement(s). Janie needs the box to hold a minimum of 700 in^3 of beads and

d. Justify whether or not Janie’s box meets the minimum storage requirement(s). e. Sketch a rough graph of the polynomial function. Part B Janie needs to create an open top storage box for a doll that she made out of buttons and beads. She has a 24 inch sheet of cardboard to create the box. a. What is the maximum volume of the box she can create? b. She needs the volume to be at least 1000 in^3. Will she be able to create a box with the needed volume from the material she has? Justify your response.	buttons and Janie determines that the box she found at the thrift store will hold 702 in^3 of material. **(2pts)** e. zeros: 0, 3, -4 **(3pts)** intervals: $x < -4$, $-4 < x < 0$, $0 < x < 3$, $x > 3$ **(4pts)** $f(-6) = -108$ **(1pt)** $f(-2) = 20$ **(1pt)** $f(1) = -10$ **(1pt)** $f(5) = 90$ **(1pt)**  **(graph 1pt)**

	Part B a. $V(x) = x(24 - 2x)^2$ **(1pt)**  24 inches **(2pts)** Maximum Volume is 1,024 in^3 **(1pt)** b. Yes. Janie will be able to construct the box she needs. The 24 square inch piece of cardboard will make an open top box with a maximum volume of 1,024 in^3. Janie needs the box to have a volume of at least 1,000 in^3. The material will meet her needs. **(2pts)** **Performance Based Assessment Task** **Total Points: 27**

MISSISSIPPI DEPARTMENT OF EDUCATION

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Lesson Plan Attachments

Mississippi College- and Career-Readiness Standards
Mathematics
Real-World Context – Graph – Function

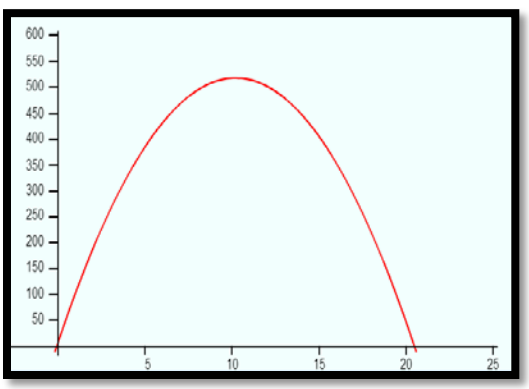
Directions:
Match the real-world context to the polynomial graph. After you have matched each real-world context with a graph, match each function to its corresponding graph.

Real-World Context

1. Models the speed of a swimmer doing the breast stroke during one complete stroke.	2. Models the average amount of oranges (in pounds) eaten per person each year in the US from 1991 to 1996.
3. Models the average price of a gallon of gas for each month from October to June.	4. Models a projectile fired into the air from a height of 8 feet above the ground.

Graphs

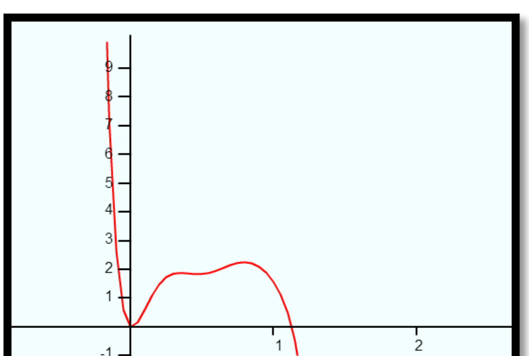
A.



Situation # _____

Function # _____

B.



Situation # _____

Function # _____

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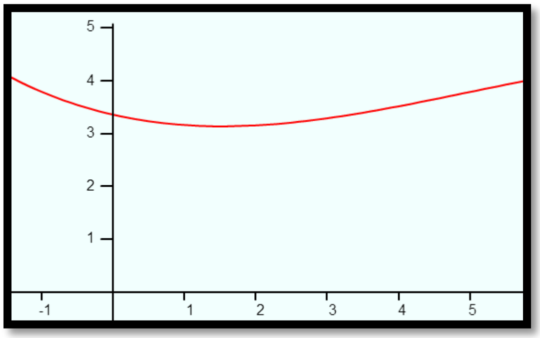
Mississippi College- and Career-Readiness Standards
Mathematics
<u>Real-World Context – Graph – Function</u>

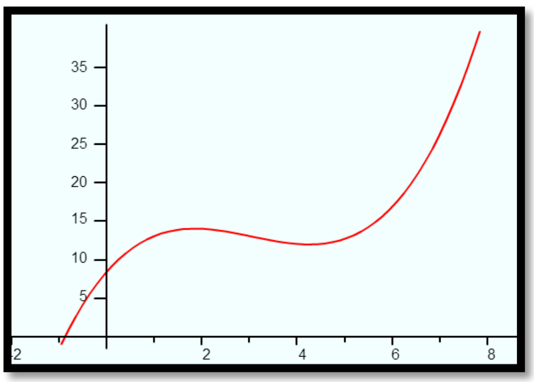
C.



Situation # _____

Function # _____

D.



Situation # _____

Function # _____

Functions

5. $f(x) = 0.298x^3 - 2.73x^2 + 7.05x + 8.45$
6. $f(x) = -4.9x^2 + 100x + 8$
7. $f(x) = - 0.0080556x^3 + 0.11881x^2 - 0.30671x + 3.36$
8. $f(x) = -241x^7 + 1062x^6 - 1871x^5 + 1647x^4 - 737x^3 + 144x^2 – 2.432x$

Attachment #2

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Mississippi College- and Career-Readiness Standards
Mathematics
Special Cases Review

Directions:
With a partner, list as many characteristics for each Factoring Technique or Special Factoring Case. Complete this activity without any references. After 7 minutes, use your class notes to add any additional information.

Difference of Two Squares	Quadratic Pattern	Factor by Grouping

Attachment #2

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Mississippi College- and Career-Readiness Standards
Mathematics
Special Cases Review

Sum of Cubes	Difference of Cubes	Polynomial Division

Mississippi College- and Career-Readiness Standards

Mathematics

Factoring Polynomial Expressions

Directions:

Factor each polynomial expression into linear factors. You may stop if a quadratic factor yields complex solutions. You may use your ***Special Cases Review*** to assist you as needed.

1. $\dfrac{2x^3 - 3x^2 - 18x - 8}{x - 4}$

2. $x^3 + 27$

3. $81x^4 - 625$

4. $x^3 + 4x^2 - 16x - 64$

5. $x^4 - 7x^2 - 12$

6. $8x^3 - 64$

7. $16x^4 - 1296$

8. $x^3 + 3x^2 - x - 3$

9. $\dfrac{x^3 - 4x^2 - 9x + 36}{x + 36}$

10. $x^4 - 27x^2 + 50$

Mississippi College- and Career-Readiness Standards
Mathematics
Factor and Find the Zeros

Directions:
Factor each polynomial equation into linear factors. Use the Zero Product Property to find the zeros. List the zeros and the solutions of $f(x) = 0$ for each equation on the line provided

1. $f(x) = x^3 - 5x^2 + 3x + 9$ *where* $(x + 1)$
zeros: ____________________________________
solutions: ________________________________

2. $x^3 + 6x^2 - 4x - 24 = 0$
zeros: ____________________________________
solutions: ________________________________

3. $f(x) = (x^2 + 2x - 15)(x^2 - 12x + 27)$
zeros: ____________________________________
solutions: ________________________________

4. $x^3 + 5x^2 - 16x - 80 = f(x)$
zeros: ____________________________________
solutions: ________________________________

Enrichment:
For the polynomial equation provided below, $(x - 2)$ and $(x - 4)$ are linear factors. Find an additional linear factor. Find all five solutions. (Note: solutions may be real or complex.)

$$X^5 - 6x^4 + 8x^3 - 8x^2 + 48x - 64 = 0$$

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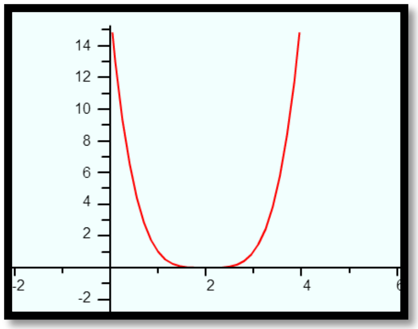
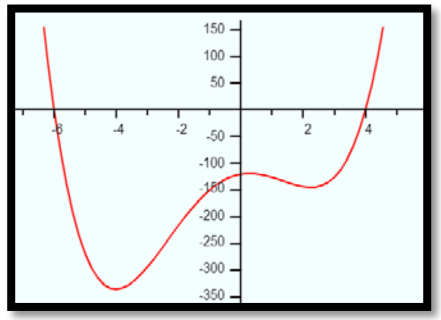
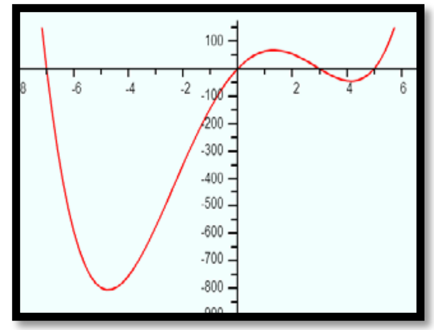
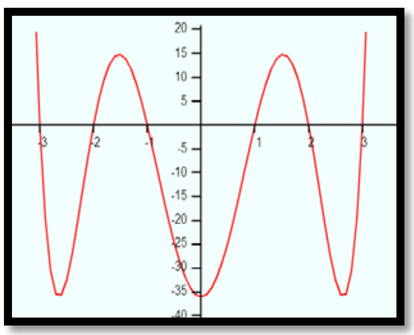
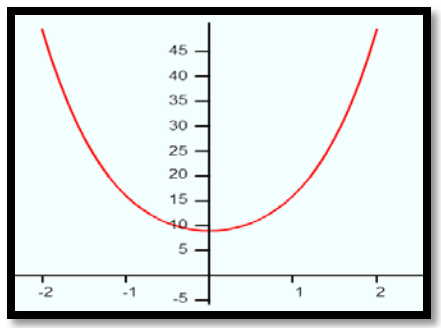
Mississippi College- and Career-Readiness Standards
Mathematics
Where's My Zero, Where's My Graph?

$f(x)$ = $(x^2 – 4x + 4)(x – 2)(x – 2)$	**$f(x)$ = $(x^2 + 5)(x^2 + 2x – 24)$**	**$f(x)$ = $(x^2 – 5x)(x^2 + 4x - 21)$**
$f(x)$ = $(x^2 – 1)(x^2 – 4)(x^2 – 9)$	**$f(x)$ = $(x^2 + 3)^2$**	**Zeros: -6, 4** **Solutions to $f(x)$ = 0: -6, 4** **x-intercepts: -6, 4**
Zeros: -7, 0, 3, 5 **Solutions to $f(x)$ = 0: -7, 0, 3, 5** **x-intercepts: -7, 0, 3, 5**	**Zeros: -1, 1, -2, 2, -3, 3** **Solutions to $f(x)$ = 0: -1, 1, -2, 2, -3, 3** **x-intercepts: -1, 1, -2, 2, -3, 3**	**Zeros: 2, 2, 2, 2** **Solutions to $f(x)$ = 0: 2** **x-intercept: 2**

Mississippi College- and Career-Readiness Standards
Mathematics
<u>Where's My Zero, Where's My Graph?</u>

		
		**Zeros: none** **Solutions to *f(x)* = none** ***x*-intercepts: none**

Attachment #6

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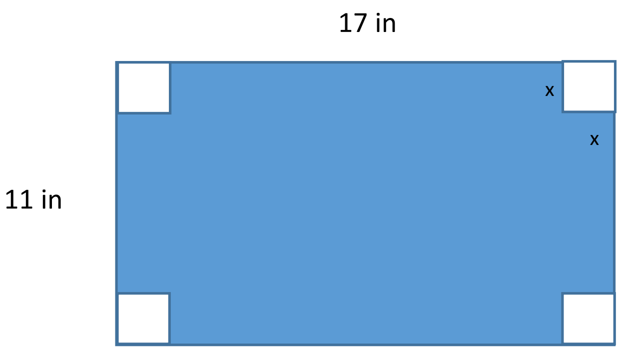
Mississippi College- and Career-Readiness Standards
Mathematics

Open Top Box Activity

Directions:

1. Form a group of four and create an open top box from an 11 x 17 sheet of paper.
2. Cut out congruent squares of your choice from each corner and fold in the sides to create the open-topped box.
3. Record the box’s measurements and calculate the volume.
4. Record the volume in the table.
5. Each group will record its measurements to the table below.
6. The class will discuss which group created a box with the maximum volume.
7. Answer the questions on page 2.



l = _________ w = _________ h = _________ V = _________

Group	Length	Width	Height	Volume
1				
2				
3				
4				
5				
6				
7				
8				
9				

Which group(s) created the box with the maximum volume? ___________________

Mississippi College- and Career-Readiness Standards
Mathematics
Open Top Box Activity

1. What type of polynomial function could be used to model this data?

2. Are there any clues in the maximization formula that can help with question 1?

3. Use a regression technique to find the function that can be modeled by the data. Does a quadratic or cubic regression best fit the data? Explain.

4. Find the maximum volume of the box.

5. What are the dimensions of the square that should be cut out from each corner of the paper to maximize the volume?

6. What are the final dimensions of the box that yield the maximum volume?

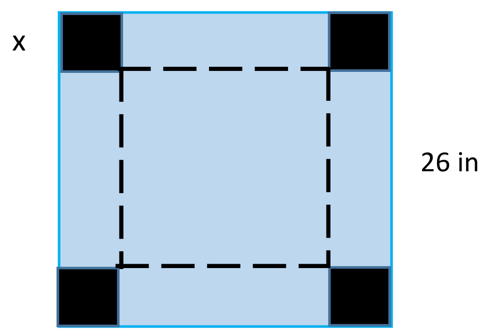
7. How does the possible values for the height of the box relate to the domain of the volume function?

8. Is the maximum volume always the best option for constructing a box?

Mississippi College- and Career-Readiness Standards
Mathematics
Modeling Polynomial Functions

Directions:
Complete the activity as indicated. Write the equation represented by the real-world problem. Use graphing technology to find the maximum or minimum for the problem.

1. a. Construct an open-top box that has a maximum volume that can be created from a square with a side length of 26 inches cut from each corner.
   b. Identify the equation that can be represented by this problem.
   c. Identify the maximum and/or minimum.



2. Bluebird Airline Company is redesigning their overhead storage bins. The new design allows for bags to have a length 12 inches greater than their depth. Each bag must meet the original design regulations which dictate the sum of the length, width, and depth may not exceed 40 inches. What is the maximum volume of an allowable piece of luggage?
3. Susan's Specialty Company makes wristlets for young adults. The production cost can be found using the equation $C(x) = 0.04x^2 - 6.405x + 22506$. If x represents the number of wristlets in hundreds, find the number of wristlets that should be manufactured to minimize production costs.

Modeling Polynomial Functions

4. A rocket is launched upward from a platform 20 feet above the ground with an initial velocity of 112 feet per second. Its height, *h,* after *t* seconds can be found by the equation $h(t) = -16t^2 + 112t + 20$. Assume air resistance is neglected.

   a. How long will it take, *t,* for the rocket to return to the ground?

   b. After how many seconds will the rocket be 100 feet above the ground?

   c. How long will it take the rocket to reach its maximum height?

   d. What is the maximum height, *h*, of the rocket?

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Mississippi College- and Career-Readiness Standards
Mathematics
Frayer Model -Polynomials

Factoring Techniques

Characteristics of Graph

Polynomial Function

Examples

Non-examples