Exemplar Lesson Plan

“Fraction Division-Going Beyond the Algorithm”

6.NS.1
6.NS.6c
6.EE.5
6.EE.6
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## Grade: 6

### Title: Fraction Division—Going Beyond the Algorithm

### Estimated Duration: 3 Days

### Real World Purpose:
What would you do if you had a portion of a piece of cake and were required to split it into smaller parts and share it with your closest friends? How would you ensure that each resulting piece was equal in size? Fraction division allows us to do just that.

### I Can:

**6.NS.1:** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.  
*For example, create a story context for \((2/3) \div (3/4)\) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that \((2/3) \div (3/4) = 8/9\) because \(3/4\) of \(8/9\) is \(2/3\). (In general, \((a/b) \div (c/d) = ad/bc\).)* How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

**6.NS.6c:** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.  
*Note: Only the first part of this standard will be addressed in this lesson plan.*

**6.EE.5:** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.  
*Note: Only equations will be addressed in this lesson plan.*

**6.EE.6:** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
Prerequisite Skills:  (Fractions and The Number Line)

- Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$.  \(3.NF.1\)
- Represent a fraction $a/b$ on a number line diagram by marking off $a$ lengths $1/b$ from 0. Recognize that the resulting interval has size $a/b$ and that its endpoint locates the number $a/b$ on the number line. \(3.NF.2b\)
- Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. \(3.NF.3\)
- Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. \(4.NF.2\)
- Understand a fraction $a/b$ with $a > 1$ as a sum of fractions $1/b$. \(4.NF.3\)
- Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. [For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?] \(5.NF.3\)
- Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. \(5.NF.4\)
- Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. \(5.NF.7\)

Prerequisite Skills:  (Equations)

- Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. \(4.OA.1\)
- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. \(4.NBT.6\)
- Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. \(5.NF.6\)
### Materials/Resources:
- 4-function calculator
- 5 Learning Stations
- Card Stock/Construction Paper
- Color Coded Sticky Dots
- Color Pencils
- Color Tiles
- Counters
- Dominoes
- Fraction Circles
- Fraction Tiles
- Horizontal and Vertical Number Lines
- Laminating Machine or Clear Packaging Tape
- Thin-tipped markers

### Key Vocabulary:
- Complex Fraction
- Equal Grouping
- Equal Sharing
- Equivalent
- Improper Fraction
- Mixed Number
- Multiplicative Inverse
- Reciprocal
- Visual Fraction Model

* = words defined in the MS CCR Mathematics Glossary

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### Lesson Introduction

*How students take notes during the lesson is at the teacher’s discretion.*

### Pre-Class Preparation:

(#1) Prepare a small container with each of the vocabulary words provided in “Key Vocabulary” section above. As students use these words and/or the teacher defines these words throughout the entire unit, the students and/or the teacher will place these words on the Word Wall. Suggestion: Words that were introduced in a previous grade level should be printed on a different color of card stock than words that are new to this grade level. This will assist students in seeing the learning progression of fractions—from year to year. The teacher should consider various ways to make the Word Wall interactive throughout the unit and the entire year.
(2) Set up 5 Learning Stations on the wall labeled “Problem Set A, Problem Set B, Problem Set C, Problem Set D, and Problem Set E” throughout the room. Each station will be equipped with an enlarged copy of each problem from Attachment #1, What is Your Division IQ?

(3) If the teacher does not have a horizontal and vertical number line diagram laminated and taped to each student’s desk, s/he may use the Horizontal and Vertical Number Line Diagram located in Attachment #3. The horizontal number line should be taped to the top center of each student’s desk. For left-handed students, their vertical number line should be taped to the left side of their desk; and, right-handed students should have their vertical number line taped to the right side of their desk. To meet the needs of a wide range of learners, the teacher may want to create a variety of different number lines based on a particular language, denominator, beginning or ending number, labels, increments, and whether or not a particular benchmark fraction is/is not given. The website http://www.math-aids.com/Number_Lines/ allows the teacher to do this. He/she will need to print each number line diagram on cardstock, cut them out, and laminate them. If a laminating machine is not available clear packaging tape will work also.

Student Exploration Activity:
For this student exploration activity, students are presented division problems in a variety of ways, namely a real-world context, a pictorial representation, and a symbolic representation which includes integers and a mixed number. [See Attachment #1: What is Your Division IQ?] Each problem set allows the teacher to identify any deficiencies that may exist related to division and fractions from previous grades. This problem set will allow the teacher to assess key vocabulary from the previous grade and provides an opportunity for students to demonstrate how much they do/don’t understand about division. For example, if students struggle with identifying the dividend and the divisor in a word problem that will be evident based on the response they provide in Problem Set A. For students that struggle with interpreting a picture, diagram, or other visual fraction model their deficiencies will be evident in Problem Set B. This will further indicate where the teacher needs to provide direct instruction during the lesson as students begin to model fraction division. If a student does not understand that 4 ½ = 4 + (½) and not 2, then their response on Problem Set D will shed light on a deficiency with mixed numbers. Problem Set E will help the teacher identify those students that may have only seen the fraction bar in a horizontal format. This problem set will also forecast this unit by seeing which students have already learned the “shortcut algorithms” commonly known as “multiply by the reciprocal” -or- “keep-change-flip”. It will also allow the teacher to see which students tend to avoid problems with fractions and those that do not understand that they can simplify 6 3 prior to performing the calculation. Moreover, Problem Set E will also provide an early indication of those students that equate dividing by ½ with cutting something in half.
Teacher Instructions: The teacher will provide each student with a copy of the attachment *What is Your Division IQ?* and read the directions aloud. Students will work independently on this activity as the teacher passes each student a Ziploc bag of counters and color pencils to allow them to model their solution. Technology may only be used by struggling learners to verify their solutions once they have completed each problem set. Additionally, the teacher may guide struggling learners by incorporating strategies such as:

- **Problem Set A:** Encourage the student to visualize that this problem is about the students in the classroom right now.

- **Problem Set B:** Ask the student to identify how many equal parts do they see.

- **Problem Set C:** Ask the student what does the “fraction bar” represent/mean?

- **Problem Set D:** Ask the student to create a word problem to model the expression.

- **Problem Set E:** Identify if the student can rewrite the problem in an equivalent form.

<table>
<thead>
<tr>
<th>Problem Set A</th>
<th>Problem Set B</th>
<th>Problem Set C</th>
<th>Problem Set D</th>
<th>Problem E</th>
</tr>
</thead>
</table>
| Half of the 24 students in Ms. Martinez’s class have pets. Half of the students with pets have cats. Half of the students with cats have kittens. How many students have kittens? | What does the following diagram illustrate? | \[
\frac{15}{3}
\] | \[
\frac{4}{2}
\] | \[
\frac{6}{2}
\] |
Think-Pair-Share-Report Out: After students have completed this activity the teacher will instruct them to rank each problem set in order of difficulty—with 1 being “Easy” and 5 being “Challenging.” On the teacher’s cue, students will proceed to the Learning Station that they placed a “1” beside and discuss their work and visual model with their peers. (Note: if the teacher has long tables, s/he may modify this activity by allowing students to work at each table as opposed to standing up at each station.) After 2 minutes, the teacher will ring a bell and students will proceed to the station that they placed a “2” beside. After 2 minutes, the teacher will ring the bell again and students will proceed to the station that they placed a “3” beside. This rotation will continue for the next 6-8 minutes until all students have visited each of the 5 Learning Stations. While students are at each station, the teacher should encourage them to write down any “Aha Moments” or “Bright Spots” they encounter as they listen to their peers. [The teacher can have them do this in the margins or s/he may what to have students maintain a running log of “AHA Moments/Bright Spots” in their notebook/binder throughout the unit. [See Attachment #2: “AHA Moments” and “Bright Spots”]]

As this activity continues, the teacher must keep in mind that some stations may have more students at them during one of the rotations; therefore, it is imperative that s/he ensures time on task. All students will return back to their desks and the teacher will ask for volunteers for each problem set. Students will come to the white board or overhead as needed. The teacher will pay close attention to how students use the following vocabulary words: fraction, rational number, dividend, divisor, quotient, equal grouping, sharing, and equivalent. At this point in the lesson it is very important that the teacher reinforce the difference between a fraction and a rational number as noted in the MS CCR Standards for Mathematics Glossary.

- **Fraction.** A number expressible in the form \(\frac{a}{b}\) where \(a\) is a whole number and \(b\) is a positive whole number. (The word *fraction* in the MS CCR standards always refers to a non-negative number.)

- **Rational number.** A number expressible in the form \(\frac{a}{b}\) or \(-\frac{a}{b}\) for some fraction \(\frac{a}{b}\). The rational numbers include the integers.

To put “closure” on the launch activity and ensure all students are have mastered the skills included in this activity, students will participate in a 3 minute-round of “Mystery Object Riddle”.” “Mystery Object Riddle” is when the teacher chooses a mystery object (or set of objects) in the room and makes up a riddle about it based on the concept being taught in class. For example:
Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Lesson Activities

Day 1  (Objective: To compute and interpret with unit fractions)
1.  (Review unit fractions and other familiar rational numbers)  The teacher will begin the lesson by explaining that many geometric shapes can be divided into congruent parts to reveal fractional pieces. S/he will remind students that in Grades K-2 they partitioned shapes into halves, thirds, and fourths. In Grade 3, they began representing those smaller parts into what were called unit fractions. And, s/he will further explain that in Grades 4-5, they began examining fractions with a denominator of 2, 3, 4, 5, 6, 8, 10, 12, and 100. S/he will ask students to explain what happens as the denominator gets smaller (i.e. closer to zero) and what happens as the denominator gets larger? The teacher will call out a predetermined list of unit fractions and encourage students to locate them on the numbers lines taped on their desk by using a color coded dot or thin-tip marker. [As mentioned in the Preparation Section, the teacher should prepare a variety of number lines in the event interventions are necessary. See the variety of number lines included below. [Available in Attachment #3, Supplemental Pages.]

Note: The teacher may need to “stage” the room by bringing in different objects and placing them strategically in the room. If this is the case, the teacher should not provide any clues to students about the placement of this objects but encourage them to get up and find x number of “riddle objects.”
As the teacher calls out a predetermined list of unit fractions, s/he will need to walk around and validate where students place these values on their number line and select students to discuss aloud whether each unit fraction is closer or further away from the benchmark fractions of 0, 1, and ½. When/if a student marks a value in the wrong location, the teacher can ask probing questions for example:

“What does the fraction represent?”
“How many equal parts is the number line divided into from 0 to 1?”
“What does the numerator tell us?”

The teacher may decide to call out a predetermined list of negative fractions (i.e. rational numbers), fractions that are equivalent to whole numbers, and other fractions that are not unit fractions (e.g. $\frac{6}{3}, \frac{15}{3}, \frac{0}{2}, \frac{3}{5}, \frac{5}{6}, \frac{6}{9}, \frac{5}{5}$ etc). The teacher will need to include real world contexts in a few of his/her examples as well. For example:
“James lives \(\frac{3}{4}\) of the way from school and Denise lives \(\frac{3}{9}\) of the way from school. Who lives the closest to the school? Explain your reasoning using your number line and words from the Word Wall.”

“Chef Roble and Paula Deen both have a breakfast casserole recipe online. Chef Roble’s recipe requires \(\frac{2}{3}\) of a cup of butter. Paula Deen’s recipe requires \(\frac{5}{8}\) of a cup of butter. Whose recipe uses more butter? Explain your reasoning using your number line diagram and words from the Word Wall.”

The teacher should spend sufficient time allowing students to make comparisons aloud using all four inequality signs (<, >, ≤, and ≥), the list of fractions and rational numbers called out, and the benchmark fractions of 0, \(\frac{1}{2}\), and 1. The teacher should be very strategic in selecting fractions in his/her word problems that students tend to have misconceptions about based on the size of the numerator/denominator. The teacher should also create/use real world contexts in which the fractional values are very close together or almost equivalent—as seen in the Chef Roble and Paula Deen problem above. Finally, the teacher should allow students that are performing above grade level to offer their reasoning at the front of the class on selected problems. This out-loud student modeling may assist students that are performing below grade level and in need of peer support. [Note: this activity can be modified by displaying each numerical value and real world context on the overhead/white board/or card stock.]

2. (Discuss visual fraction models) The teacher will discuss that while there are many shapes that lend themselves to fractional understanding, they will spend their time today using horizontal rectangular (visual) models. S/he will ask one of the students that may be performing below grade level to place this vocabulary term on the Word Wall and to point at each word separately (i.e. horizontal, rectangular, visual, model) and to define it. S/he will then instruct each student to close their eyes and with their fingers, “air draw” exactly what they just heard the student describe. This will create a very safe environment for all students. If a particular student does not “create” a figure that resembles this \[\text{figure}\] then the teacher may have to re-direct, but chances are s/he will not need to. The purpose of this “air drawing” is to get students familiar with the concept before it is given to them. This will make below average students somewhat comfortable with modeling fraction division; and, later, their interpretation of the quotient and the visual model.
Once this student has set down, the teacher should ask another student that may be performing below grade level to come to the white board/overhead and draw their figure. She will purposely hand that student a red or blue marker. The purpose of this is to trigger the response for the next question s/he will ask: “What does your figure remind you of?” -or- “What does this figure look like to you from elementary school?” After some time (and if needed, some prompting) the students should suggest that it looks like a Fraction Tile—which most students learned to use in their MS CCR Grade 3 class for the standards 3.NF.1, 3.NF. 2, and 3.NF.3. S/he will tape the term Fraction Tile directly to the term Horizontal Rectangular Model to signal to students that these words are synonymous. S/he will then write the number “1” in the center so that it resembles the Whole (1) unit fraction tile.

[Note: some students equate the color red with a number being negative; therefore, the teacher may want to address this or simply use the blue marker instead.] If a student was not exposed to fraction tiles in Grades 3-5, the teacher should have a full set available for demonstration purposes. The teacher will then tape the term Visual Fraction Model to the terms Horizontal Number Line and Fraction Tiles without saying a word and ask students what does that indicate to them (Answer: they are synonymous). Finally the teacher will define a Visual Fraction Model as a tape diagram, number line diagram, or an area model. And hopefully, one of the advanced students will begin a brief discussion equating these terms with the term Number Line Diagram thus prompt the teacher to tape all of these words together on the Word Wall. This Word Wall activity is important as students prepare to take standardized assessments in the coming years.

3. (Understanding the parts of a complex fraction) The teacher will point to Learning Station E where the item below was included and ask students to journal/write down in words what they know about this expression. S/he will encourage them to use the terms placed on the Word Wall today. S/he will then tell the students that this is an example of a complex fraction and place it on the Word Wall as well. Students will have 2 minutes to briefly journal/write down in their notes what they believe a complex fraction is. Sample responses may include:
“A complex fraction is a fraction in which the numerator and/or the denominator or both includes a fraction.”

“Mrs. Davis bought three large boxes and wants to split them in half so they will fit in the trunk of her car. How many halves will she need to split the boxes into in order for the boxes to fit?”
If students find that this task is challenging, the teacher may allow them to verbalize what the complex fraction is: e.g., “How many halves are there in 3 wholes?” -or- “How many halves are there in six-halves?” -or- “How many halves can fit into 3 wholes?” (Answer: 6). The important thing is to allow students to demonstrate that they understand the role of the dividend, divisor, and division.

4. **(Understand a visual fraction model and interpret the quotient for unit fraction division)** At this junction, the teacher may need to model this problem using either a number line diagram, an area model, or fraction tiles at the white board/overhead.

Green: 6 whole units divided into 2 parts
Blue: Half of the 6 whole units is 3
Yellow: there are 6 halves in each of the 3 parts

(This number line is available in Attachment #3, Supplemental pages.)

It takes 6 halves to make 3 wholes.
S/he will then give students the following problem: “Felipe has \( \frac{1}{2} \) of a pound of chocolate and decides to divide the chocolate into pieces that \( \frac{1}{4} \) pound each. How many pieces of chocolate will that require?” S/he will write the complex fraction that represents this scenario at the front of the room. Students will then journal -or- discuss how this complex fraction is similar to and different from the one provided in Learning Station E. In their response, students must include the similarities and differences by: (1) looking at each fraction separately, (2) circling and identifying the numerator and the denominator in each fraction, (3) explaining the role of each number, (4) using color pencils and a thin-tipped marker to create a visual fraction model, and (5) justifying their work by writing a brief description of each component in the margin. During this time, the teacher will walk around to assess student understanding and provide guiding questions as needed. A student’s work sample may look like this:
Variations for Instruction: Teachers might consider (1) allowing students with learning disabilities and those performing below grade level to use unlined paper to create their visual fraction models for this part of the lesson. Notebook paper has lines that may be distracting for special populations, or (2) using transparencies for those students that have difficulty understanding that the whole must remain the same when portioning shapes by allowing them to trace the original whole (one). This will help them develop a better understanding of this concept. Or, (3) consider purchasing Fraction Tiles that are multi-colored instead of a singular color. This will especially helpful for visual learners. Students create their own Fraction Strips (with or without labels) by accessing [http://www.math-drills.com/fractions.shtml](http://www.math-drills.com/fractions.shtml).

While at the front of the room, the teacher will provide several more examples involving unit fractions in the form $1/b$ as the dividend and as the divisor, and repeat the process outlined in this section. Attachment #4 Dividing Unit Fractions includes additional problems that can be used for classroom use. S/he will encourage students to use visual fraction models to justify all of their work. Below grade level students may use a calculator to verify their answers after they have completed all problems by hand and drawn a visual fraction model.

*Homework may be assigned through the use of Essential Questions at the teacher’s discretion.*
Day 2  (Objective: To compute, interpret, and discover the “algorithm” for dividing fractions)

Before class starts the teacher will (1) place a “Strongly Agree” and “Strongly Disagree” voting sign under each student’s desk. [See Attachment # 5 Strongly Agree, Strongly Disagree Voting Signs.]  Students will use these signs periodically during the lesson today.  
(2) Develop a classroom list which groups students into teams of four (1-above grade level student, 2-on grade level students, and 1-below grade level student) and post this list on the classroom door.

1.  (Understanding a visual fraction model to interpret the quotient for non-unit fraction division)  The teacher will display the following visual fraction models at the front of the room and have students write a formal description of them, and to locate and label each fraction on the vertical and horizontal number line diagrams taped to their desk. (The teacher may instruct them to use only one number line if they choose.)

![Visual Fraction Models]

The teacher will walk around to ensure that students are using the appropriate vocabulary in their written description of each model and have plotted each fraction in the correct location on their number line diagrams. S/he will ask for volunteers to read what they have written and s/he will prompt students to make sure that they keep in mind that the size of whole (1) is the same. She will then display the following number line at the front of the board and instruct two students to come up and demonstrate where each value lies on the diagram.  (Note: This number line should be chosen because it allows the teacher to gauge whether students use the benchmark fraction of ½ as a reference point and if students remember to partition the diagram into thirds and/or sixths based on the models above. If this does not occur, the teacher should ask guiding questions to get students to do so).  Student responses should resemble:
At this time, the teacher will have students locate their *Strongly Agree, Strongly Disagree Voting Signs* under their desk, and on cue they will discuss the placement of each fraction in relationship to 0, ½, 1, and each other. The teacher will select several students to justify their “vote”.

2. *(Making predictions about the quotient)* The teacher will instruct students to copy the following prompt and to complete each sentence using the previous fractions: “The fraction ____________ is closer to the number 1, therefore it is ___________________. The fraction ________________ is closer to zero, therefore it is ___________________.” S/he will then instruct students to write two different complex fractions using the two fractions above. Students responses should be:

\[
\frac{2}{3} \div \frac{1}{6}
\]

and

\[
\frac{1}{6} \div \frac{2}{3}
\]

3. **THNK-WRITE-GROUP-SHARE:** The teacher will give students 3 minutes to review each complex fraction and to write a prediction as to which problem will have the greatest quotient (solution). Students will also be encouraged to justify their thinking. The teacher will ring a bell to cue students to get into their predetermined teams of four and to discuss their predictions. During this time, the teacher will encourage them to use the terms “dividend” and “divisor”. As students work in small groups, the teacher will identify which groups used a visual fraction model to calculate the quotient. S/he will politely ask the group if s/he can share their work with the entire class by displaying it at the front of the room. The group/student that created the work will describe their thought process with the
entire group. And, at key points during the students’ explanation, on cue, students will use their *Strongly Agree, Strongly Disagree Voting Signs* to identify whether or not they agree with the statements being made and the work presented. The teacher will indicate that the next activity will allow them to explore this concept even further. S/he will then pass out a small Ziploc bag of dominoes and the handout *Predict and Discover* to each group. [See Attachment #6.]

For this activity, students will explore whether or not their predictions were correct. They will also predict whether the quotient for any complex fraction will be greater than or less than 1 based on the numerical relationship between the dividend and divisor. Problems #4 and #5 will allow the student to discover what happens when both the dividend and divisor of a complex fraction have the same numerator -or- denominator. Problems #6, 7, and 8 allow them to use dominoes to create their own complex fractions. And, the remaining 3 problems will allow students to observe the relationship between the dividend and the divisor and make a conjecture about a shortcut (“algorithm”) for finding the quotient. Students will have an estimated 30-40 minutes to complete this activity in their groups. After time is called, the teacher will identify which problem each team will display on chart paper. Students may also use their *Strongly Agree, Strongly Disagree Voting Signs* to initiate classroom discourse.

Note: The teacher should be very strategic about which dominoes they place in each bag. S/he will want to consider using some dominoes that are not a unit fraction, those that cannot be reduced to a whole number, and those that cannot be modeled by Fraction Tiles/Circles. This will be a productive struggle for some students, but the intent is to get them to rely on their conceptual understanding of fractions to respond.

4. (Class presentations) After each problem has been presented, the teacher will re-emphasize the purpose of division and that the shortcut “algorithm” should not take the place of developing a visual fraction model and being able to interpret it—especially in a real world context. The teacher may encourage all students to turn their dominoes 180° to “see” the reciprocal of the fraction in the denominator (i.e. the divisor). This will be extremely helpful for visual learners.
Always ask yourself if your answer is reasonable? Why/why not?

*Homework may be assigned through the use of Essential Questions at the teacher’s discretion.*
Day 3: (Objective: To create word problems/real-world contexts)

1. (Re-iterating “Reasonableness”) The teacher will re-emphasize the importance of students asking themselves if the answer (quotient) they have provided is reasonable. S/he will spend a few minutes at the beginning of class ensuring that students understand conceptually why it makes sense that if the divisor is smaller than the dividend then the quotient will be greater than 1.

\[
\frac{5}{6} = \frac{10}{12} = \frac{5}{6} = 1\frac{2}{3}
\]

WHY IS MY ANSWER REASONABLE?
Because it makes sense that I can fit something smaller into something larger at least once! So my quotient must be GREATER than 1.

If the divisor is larger than the dividend then the quotient will be less than 1. S/he will provide the following examples for illustrative purposes.

\[
\frac{1}{2} = \frac{6}{10} = \frac{3}{5}
\]

WHY IS MY ANSWER REASONABLE?
Because it makes sense that I can’t fit something larger into something smaller not even once! So my quotient must be LESS than 1.
2. *(Creating Real World Problems)* For many students, completing word problems is a challenging task. And, asking students to create their own word problems which includes fractions can be even more challenging. So for this activity students will use the pictures of everyday objects to motivate them as they create 8 unique, real-world problems that require fraction division. *(Estimated Time: 45 minutes)*

3. *(Class presentations and Gallery Walk)* After students have completed their “story book”, the teacher will ask for student volunteers to come up and share one of their “stories”/problems with the entire class. Students that are performing below grade level should especially be encouraged to share. After a few students have shared their work, the teacher will instruct all students to post their storybooks on the walls around the room. The teacher will then pass each student 2 Post-it Notes. With approximately 10 minutes remaining in the class, students will participate in a Gallery Walk and locate 2 storybooks to leave a comment or question on. Students will have the opportunity to review the Post-it Notes left on their work and take them home for review. The teacher will address questions as needed prior to closing out the lesson.

*Homework may be assigned through the use of Essential Questions at the teacher’s discretion.*
Lesson Closure

**Day 1:** Students will write 3 big ideas that they learned from today’s lesson on one of their light bulbs and share it with a classmate. The classmate must initial beside the light bulb acknowledging the discussion and write one question for the student to consider for homework. The teacher will select several students to share with the class and respond as appropriate.

**Day 2:** Students will respond to the prompt below and justify their response appropriately using visual fraction models.

**Essential Questions:**
- Is division related to “repeated subtraction” as multiplication is related to “repeated addition”?
- Does the order in which you divide two fractions matter? Justify.
- What is the relationship between the term “complex fraction” and “fraction division”?
- When are Fraction Circles a more appropriate tool to use than Fractional Tiles?
- Can Fractional Tiles be used to model all fraction division problems?
- How is dividing whole numbers similar to dividing fractions?
- What is the relationship between division of whole numbers and multiplication of fraction reciprocals?
- How can a whole number be divided by a fraction?
- Why and how can an area model, a number line, or fraction tiles be used to model fraction division?
- What is the relationship between multiplication by a fraction and division?
- How can division of fractions be used to represent and understand real-world, and mathematical problems?
- Why must the size of the whole (1) be the same when evaluating the numerator and denominator of a complex fraction?
- How can the proximity of 1 or 0 to a given fraction be used to help make predictions when simplifying complex fractions.
**Day 3:** Each student will receive the following prompt and are required to use the information they learned over the past few days to respond. Students must provide a rationale and/or a visual fraction model to justify their response.

Determine the **missing values** for each complex fraction listed below.

\[
\begin{align*}
\frac{3}{2} & \times \frac{9}{8} = \frac{\text{?}}{8} \\
\text{?} & \div \frac{6}{7} = \frac{1}{7} \\
\frac{8}{5} & = \frac{16}{\text{?}}
\end{align*}
\]
Standards for Mathematical Practice

✓ Make sense of problems and persevere in solving them.
✓ Reason abstractly and quantitatively.
✓ Construct viable arguments and critique the reasoning of others.
✓ Model with mathematics.
✓ Use appropriate tools strategically.
✓ Attend to precision.
✓ Look for and make use of structure.
✓ Look for and express regularity in repeated reasoning.

Supplemental Activities

Intervention

- **Attachment #8 Intervention #1: Fractional Parts** gives struggling learners an opportunity to revisit labeling, constructing, and identifying fractional parts.

- [http://www.commoncoresheets.com/Fractions.php](http://www.commoncoresheets.com/Fractions.php) Allows the teacher to customize worksheets for students that are having difficulty locating fractions on a number line.

- **Attachment #9 Intervention #2: RWPs Dividing Unit Fractions and Whole Numbers** gives students extra practice with real world problems (RWPs) that are practical and present a lower-degree of difficulty.

Enrichment

- Have student explore how the use of common denominators may be useful when dividing fractions. Give the student clear guidance on how to prepare “student-friendly” notes and allow them to present their findings to the class using examples of problems presenting during the lesson. Include this activity at the latter part of Day 2 or Day 3.

- **Attachment #11 Enrichment #1: Divide 3 Fractions** contains 9 problems where advanced students may attempt to divide 3 fractions at the same time.
• **Attachment #10 Intervention #3: Fraction Puzzles** allows the student to work on fraction division with unit fractions, whole numbers, and visual models.

• **Attachment #12 Enrichment #2 Funny Fractions** includes funny cartoons on fractions. Allow the student to create a similar 8 ½” x 11” collage on card stock/construction paper of funny cartoons using many of the concepts taught in this unit.
### Math Task

**Task Available at**

www.illustrativemathematics.org

You are stuck in a big traffic jam on the freeway and you are wondering how long it will take to get to the next exit, which is 1 1/2 miles away. You are timing your progress and find that you can travel 3/4 of a mile in one hour. If you continue to make progress at this rate, how long will it be until you reach the exit? Solve the problem with a diagram and explain your answer.

Since 1 1/2 = 3/2 and it takes an hour to travel $\frac{3}{4} = \frac{3}{2}$ miles, we can look at the number lines above and see that it will take $2 \frac{1}{2}$ hours to travel the distance to the exit.

Since we are asking "How many $\frac{3}{4}$ are there in 1 1/2?" this is a "How many groups?" division problem:

$$\frac{1}{2} \div \frac{3}{4} = ?$$

We have found that the answer to this division problem is $2 \frac{1}{2}$.

### Rubric/ Plausible Student Response(s)

- **Sample:**
  - Visual Fraction Model (15 points)
  - Number line partitioned correctly (10 points)
  - Correct solution (25 points)
  - Reasoning (25 points)
  - Interpretation of solution (25 points)
Lesson Plan
Attachments
What Is Your Division IQ?

Materials Needed:
- counters
- buttons
- colored pencils

Directions:
Review and solve each problem set below. Respond to each item based on the information that is given. Use either counters, button, or colored pencils to justify your response. Be sure to include as many vocabulary words you may have learned in 5th grade.

<table>
<thead>
<tr>
<th>Problem Set</th>
<th>Describe your solution in words</th>
<th>Model Your Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Set A</td>
<td>Half of the 24 students in Ms. Martinez’s class have pets. Half of the students with pets have cats. Half of the students with cats have kittens. How many students have kittens?</td>
<td></td>
</tr>
<tr>
<td>Problem Set B</td>
<td>What does the following picture illustrate?</td>
<td></td>
</tr>
</tbody>
</table>

Attachment #1
<table>
<thead>
<tr>
<th>Problem Set C</th>
<th>15/3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Set D</td>
<td>(4\frac{1}{2})</td>
</tr>
<tr>
<td>Problem Set E</td>
<td>(6\frac{2}{1})</td>
</tr>
</tbody>
</table>

AHA Moments and Bright Spots

Materials Needed:
- Pencil/pen
- Color pencils (for diagrams)

CONCEPT: ______________________

Directions: Keep a log of your “AHA moments” and “bright spots” in/around each light bulb as you learn, hear, or see something that has made this concept easier to understand.
**Modifications and Uses:** *(for teacher use only)*

There are many modifications and uses for the “light bulb” template other than having students keep a personal log of their “AHA Moments and Bright Spots” in their notebook/binder. A few suggestions are provided below. On each line, write a brief description on how you could tailor the “light bulb” concept to fit the modification/use listed.

1) Targeted Intervention: __________________________________________________________
______________________________________________________________________________

2) Classroom Participation: _______________________________________________________
______________________________________________________________________________

3) Closure: _____________________________________________________________________
______________________________________________________________________________

4) Informative Assessment: _______________________________________________________
______________________________________________________________________________

5) Real-World or Cross Curricular: ________________________________________________
______________________________________________________________________________

6) Student Work Samples: _______________________________________________________
______________________________________________________________________________

7) Prior Knowledge: __________________________________________________________________
______________________________________________________________________________
Horizontal and Vertical Number Line Diagrams

Materials Needed:
- scissors
- laminating machine or clear package tape
- thin-tipped marker or color coded dots

Directions: The horizontal number line should be taped to the top center of each student’s desk. For left-handed students, their vertical number line should be taped to the left side of their desk, and right-handed students should have their vertical number line taped to the right side of their desk. Have students find and position integers and other rational numbers on a horizontal or vertical number line diagram.
Note:
This page serves as a supplement for Attachment #3 and should be used for students that prefer to use a number line from [0,10] and need to visually see the benchmark fractions 0, 1/2, and 1.
Note:
These pages serve as a supplement for Attachment #3 and should be used for students that have difficulties using the number lines from [-5,5]. These number lines may also be used for students that may or may not have difficulty visualizing the benchmark fractions 0, 1/2, and 1.
Note:
These pages serve as a supplement for Attachment #3 and should be used for students that have difficulties using the number lines from [-5,5]. These number lines may also be used for students that may or may not have difficulty visualizing the benchmark fractions 0, 1/2, and 1.
Note:
These pages serve as a supplement for Attachment #3 and should be used for students that have difficulties using the number lines from [-5,5]. These number lines may also be used for students that may or may not have difficulty visualizing the benchmark fractions 0, 1/2, and 1.
Note:
These pages serve as a supplement for Attachment #3 and should be used for students that have difficulties using the number lines from [-5,5]. These number lines may also be used for students that may or may not have difficulty visualizing the benchmark fractions 0, 1/2, and 1.
Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Dividing Unit Fractions

Materials Needed:
- Pencil
- Fraction Tiles -or- Color Pencils and Unlined Paper -or- Number Line

Directions: Find the quotient of each problem listed below. **Circle** the dividend and **underline the divisor** in the problem. Attach your scratch work showing a visual fraction model to justify your solution.

1) \( \frac{1}{8} \div \frac{1}{5} = \)
2) \( \frac{1}{8} \div \frac{1}{8} = \)
3) \( \frac{1}{7} \div \frac{1}{7} = \)
4) \( \frac{1}{9} \div \frac{1}{4} = \)
5) \( \frac{1}{6} \div \frac{1}{6} = \)
6) \( \frac{1}{6} \div \frac{1}{8} = \)
7) \( \frac{1}{2} \div \frac{1}{8} = \)
8) \( \frac{1}{3} \div \frac{1}{8} = \)
9) \( \frac{1}{7} \div \frac{1}{7} = \)
10) \( \frac{1}{5} \div \frac{1}{5} = \)
11) \( \frac{1}{6} \div \frac{1}{8} = \)
12) \( \frac{1}{5} \div \frac{1}{9} = \)
Mississippi College- and Career-Readiness Standards
Mathematics
Exemplar Lesson Plan

Strongly Agree, Strongly Disagree Voting Signs

Materials Needed:  (TEACHER ONLY)
- 2 different color cardstock
- laminating machine -or- clear packing tape
- scissors

Teacher Directions: Print the “Strongly Agree” voting signs on green cardstock. Print the “Strongly Disagree” signs on red cardstock. Laminate each sign and cut them out. Place both signs under each students desk before class begins. Instruct students to leave them under the desk for future use. (Note: teachers may decrease the size of the sign to 4” x 6”.)

Student Directions: During the group discussion, you will hold up one of the voting signs below indicating whether you agree or disagree with the information stated. Be prepared to justify your “vote”.

---

Strongly Agree
Strongly Disagree
**Mississippi College- and Career-Readiness Standards**  
**Mathematics**  
**Exemplar Lesson Plan**

**Predict and Discover**

**Required Materials:**
- Dominoes

**Optional Materials:**
(Students may choose which item(s) listed below would assist them in completing each problem.)
- Color pencils
- Construction paper
- Transparencies and Thin-Tip Marker
- Fraction Tiles
- Fraction Circles
- Number Line Diagrams

**Directions:** In teams of four, review each problem set below. Complete each item as indicated. Be prepared to share your work on chart paper for any given problem or problem set.  
(Time: 30-40 minutes)

| Problem Set | Make a prediction:  
*Will the quotient be less than or greater than 1? Justify your reasoning. Use the appropriate vocabulary terms in your response.* | Model your solution and identify your quotient |
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>#1. Given</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[
\begin{array}{c}
\frac{2}{3} \\
\frac{1}{6}
\end{array}
\]
| | Was your prediction correct? |
|  | Y | N |
| #2. Given | | 
\[
\begin{array}{c}
\frac{1}{2} \\
\frac{5}{6}
\end{array}
\]
<p>| | Was your prediction correct? |
|  | Y | N |</p>
<table>
<thead>
<tr>
<th>#3.</th>
<th>Given</th>
<th>Was your prediction correct?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Fraction" /></td>
<td></td>
<td>Y</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>#4.</th>
<th>Given</th>
<th>Was your prediction correct?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Fraction" /></td>
<td></td>
<td>Y</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>#5.</th>
<th>Given</th>
<th>Was your prediction correct?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Fraction" /></td>
<td></td>
<td>Y</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>#6.</th>
<th>Select two dominoes from the bag and create a complex fraction.</th>
<th>Was your prediction correct?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Dominoes" /></td>
<td></td>
<td>Y</td>
</tr>
</tbody>
</table>
#7. Select two dominoes from the bag and create a complex fraction.

<p>| | |</p>
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#8. Select two dominoes from the bag and create a complex fraction.

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**Was your prediction correct?**

- Y
- N

#9. List the dividend, divisor, and quotient for problems #1-8 on the lines provided below.

<p>| | |</p>
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#10. Examine each set of numbers above. Identify any relationships you see between the dividend, divisor, and quotient. Can you come up with a procedure (“shortcut”) for finding the quotient?
**Test Your Procedure/Shortcut:** Compute the quotient of each complex fraction provided below using your reasoning in problem #10. Use a calculator to verify your response.

\[
\frac{\frac{6}{9}}{\frac{1}{5}}
\]

\[
\frac{\frac{3}{5}}{\frac{\frac{3}{7}}{\frac{11}{7}}}
\]

\[
\frac{\frac{2}{3}}{\frac{\frac{11}{7}}{\frac{6}{5}}}
\]

Create your own complex fraction.
Real World Problem Starters

Materials Needed:
- Card stock or construction paper
- Scissors
- Glue
- Stapler
- Lined paper

Directions: Using a folded sheet of card stock or construction paper, cut along one of the folds to create a “story book” as shown in the figure below. Staple along the center fold to secure the story book in place. Glue a piece of lined notebook paper to the bottom center of each section, and label them from #1-8. Use the pictures on the next two pages to assist you in creating 8 unique, real-world problems that require fraction division. Cut and paste the pictures that you will use for each problem and use lined notebook paper to record your work. You may use more than one picture per problem.

Have fun and be creative.

Is your answer reasonable in the context of the problem?

Your challenge is that half of your word problems must meet the following criteria:
- Cannot use the benchmark fractions 0, 1, or ½.
- Must contain an improper fraction.
- Quotient must be reduced to a simpler form.
- Quotient must be less than 1.
Intervention #1: Fractional Parts

Materials Needed:
- color pencils
- yellow highlighter and green highlighter

Directions: Respond to each item based on the information that is given. Write your numerical response on the line. Use the margin to explain your response for each item. Highlight your numerator in yellow. Highlight your denominator in green.

What is the Fraction of the Shaded Area?

1) __________ 6) __________
2) __________ 7) __________
3) __________ 8) __________
4) __________ 9) __________
5) __________ 10) __________

Shade the Figure with the Indicated Fraction.

11) __________ 16) __________
12) __________ 17) __________
13) __________ 18) __________
Intervention #2: RWPs Dividing Unit Fractions and Whole Numbers

Materials Needed:
- pencil

Directions: Read each item below. Circle the dividend and underline the divisor in the problem. Use the space provided to create a visual fraction model to justify your solution.

1. Walker collected 1/2 of a pound of strawberries. He has to divide them equally among 5 wooden baskets. How many pounds of strawberries did Walker put in each wooden basket?

2. Kate uses 2 packets of milk powder per day to feed her little baby. How many days will 1/3 of a packet of milk powder last?

3. For a family function, a florist used 1/2 of a basket of flowers to decorate 3 windows. He used the same amount of flowers for each window. How many baskets of flowers did he use for each window?

4. On the last day of exams, our teacher had 1/2 of a bundle of blank paper left. She gave the paper to her 3 students equally. How much of the bundle did every student take home?

5. An oil factory used 1/2 of a tin of peanuts to prepare 3 drums of peanut oil. How many tins of peanuts does the factory use to fill in each drum?
6. Jackson made \( \frac{1}{3} \) of a pound of milk cake. He cut the cake into pieces to share with his 3 friends. What was the weight of each piece?

7. A street baker needs 8 chocolates to paste on each case of a Chocó roll. How many chocolates will he need for \( \frac{1}{2} \) of a case?

8. Wilson has a tray of cherries. His daughter ate \( \frac{1}{2} \) of the cherries left on the tray. She ate those cherries in 3 equal parts. What fraction of cherries did she eat each time?

9. Each day, Jordan uses 5 bags of mangoes for mango shakes for her family and tenant. How much of her usual can she make with \( \frac{1}{2} \) of a bag of mangoes?

10. Paul snatched \( \frac{1}{5} \) of a pound of grapes from his fridge. He gave them equally to 2 friends. How many pounds of grapes did Paul give to each friend?
Intervention #3: Fraction Puzzles

Materials Needed:
- Color Pencils (Red, Yellow, Green, Blue)
- Color Tiles

Directions: Use your color tiles to solve each fraction puzzle. Use the next two pages and your color pencils to record your work.

Fraction Puzzle Cards Were Purchased from www.EAleducation.com
<table>
<thead>
<tr>
<th>Card #</th>
<th>Solve the card equation</th>
<th>Draw your model here</th>
</tr>
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<tbody>
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</tr>
<tr>
<td>Card #</td>
<td>Solve the card equation</td>
<td>Draw your model here</td>
</tr>
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</tbody>
</table>
**Enrichment #1: Divide 3 Fractions**

**Materials Needed:**
- pencil

**Directions:** Calculate each quotient below. Justify your response for 5 items using an area model.

1. \( \frac{1}{4} \div \left( \frac{9}{10} \div \frac{8}{9} \right) \)
2. \( \frac{9}{7} \div \left( \frac{16}{5} \div \frac{7}{5} \right) \)
3. \( \frac{4}{3} \div \frac{2}{3} \div \frac{16}{3} \)
4. \( \frac{13}{9} \div \left( \frac{13}{3} \div \frac{16}{7} \right) \)
5. \( \frac{2}{9} \div \frac{13}{9} \div \frac{7}{5} \)
6. \( \frac{10}{3} \div \left( \frac{9}{4} \div \frac{3}{7} \right) \)
7. \( \frac{3}{4} \div \frac{9}{2} \div \frac{19}{8} \)
8. \( \frac{1}{2} \div \frac{3}{2} \div \frac{4}{3} \)
9. \( \frac{4}{3} \div \frac{5}{3} \div \frac{12}{5} \)

Problem Set Available at: [www.mathdrills.com](http://www.mathdrills.com)
Enrichment #2: Funny Fractions

Materials Needed:
- Color markers and pencils
- Newspaper or magazine clippings
- Internet clip art or images
- Construction paper or card stock
- Scissors
- Tape

Directions: Review each cartoon provided in the collage below. Develop a collage similar to this by creating your own “funny fraction” cartoons. Create cartoons that include some of the key concepts taught in the unit on division of fractions.

```
© DAVE ANDERSON  www.mutlee.com
PRINCIPAL

“I’d plead the fifth, but we haven’t learned fractions yet.”

© Original Artist
Reproduction rights obtained from
www.CartoonStock.com

7.1
9

4

J.K.S.

“7/6 of the class don’t understand a word I’m saying about fractions.”

© DAVE ANDERSON  www.mutlee.com

“We found the least common denominators in math, ran a half mile in gym, and learned about eighth notes in music. If there are any fractions in today’s English lesson I’m going to scream!”

© Mark Parisi. Permission required for use.

off the mark.com  by Mark Parisi

“BELIEVE ME, IT SOUNDS BETTER THAN IT IS.”

TWO and a half MEN

© Mark Parisi. Permission required for use.
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Mathematics
Exemplar Lesson Plan

To show you how well I understood fractions, I only did half of my homework! Aren’t you proud of me!

“No, you don’t need a permission slip to learn about improper fractions.”

“So no matter what, you can’t make them fractions?! Those are numbers I can get behind!”

Did you know that 3 out 2 people have trouble with fractions?

Whew! That makes me feel so much better!

Source: All images taken from the Internet: Google search: “fraction cartoons” on May 15, 2015